Model–Based Fault Diagnosis - An Introduction

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Model–Based Fault Diagnosis, Isolation and Identification

What is Fault Diagnosis?

> Identify that a system has an imperfect behaviour caused (most likely) by a fault.

What is Fault Isolation?

> Identify location of the fault, (i.e. find out which component has become faulty.)

What is Fault Identification?

> Estimate the size and type of the fault (i.e. malfunction or chatastrophic breakdown)

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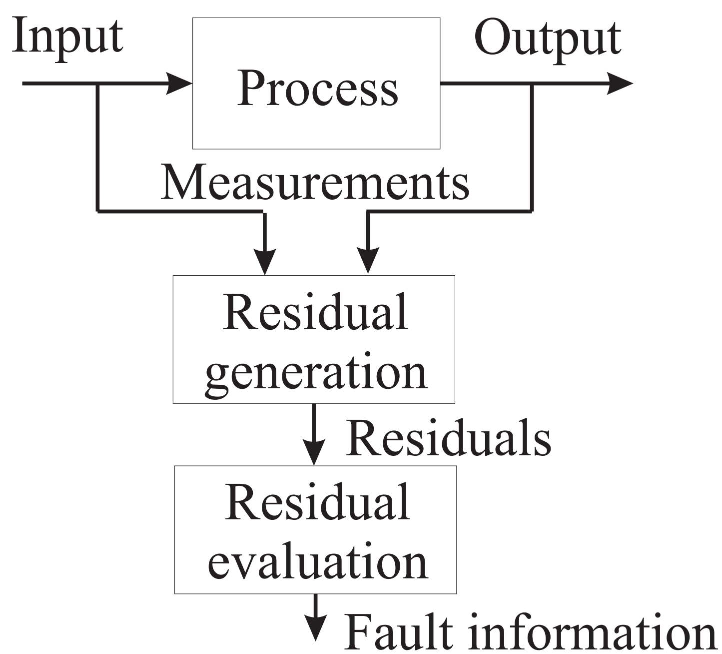
Analytical vs Hardware redudancy for FDI

- Hardware (or physical/parallel) redudancy.
  - Uses multiple lanes of senosrs/actuators/computer/software....
  - Requires extra hardware and a voting system to detect mismatch between redundant compenents.

- Analytical (functional) redudancy.
  - Uses redundant analytical relationships between measured variables.
  - No extra hardware is required.

Structure of model-based FDI system.



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Structure of model-based FDI system.

1. Residual generation.

   - This block generates residual signals using available inputs and outputs from the monitored system.
   - It should be normally zero or close to zero under no fault condition, whilst is distinguishably different from zero when a fault occurs.

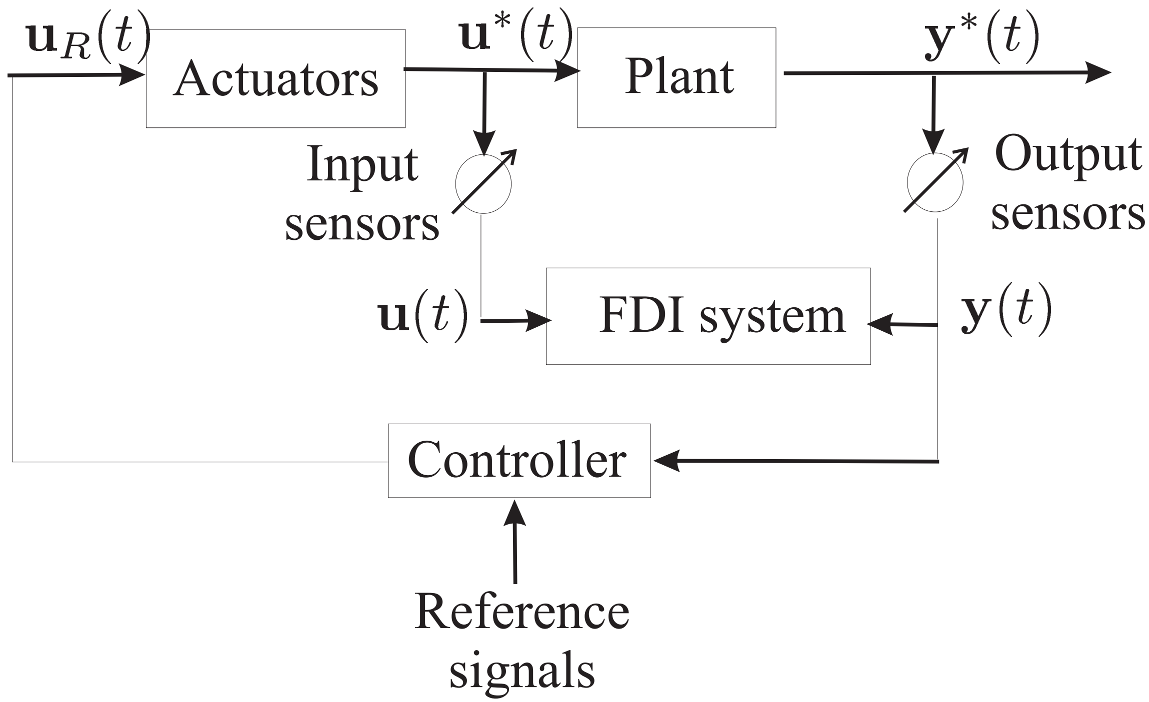
2. Residual evaluation.

   - This block examines residuals for the likelihood of faults applying a decision rule.

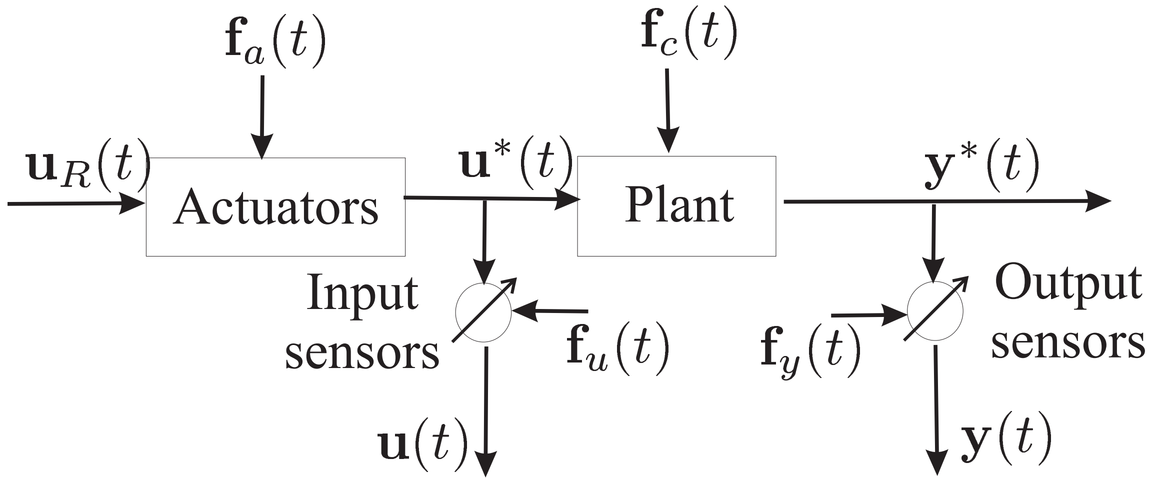
Modeling of Faulty Systems

- Open–loop system.
  - Input and output measurements are supposed completely available for FDI purpouses.
  - We do not consider the controller behaviour in the design of a fault diagnosis scheme.
- Closed–loop system.
  - Input measurements are not directly available.
  - (Robust) controller may masks fault effects.
  - Research still open.

Fault diagnosis and control loop.



The monitored system and fault topology.



Linear Plant

- Consider a state–space description of a linear plant in fault free case

$$\begin{cases} \mathbf{x}(t+1) & = & \mathrm{A}\mathbf{x}(t) + \mathrm{B}\mathbf{u}^*(t) \\ \mathbf{y}^*(t) & = & \mathrm{C}\mathbf{x}(t) \end{cases}$$

- and with faults:

$$\begin{cases} \mathbf{x}(t+1) & = & \mathrm{A}\mathbf{x}(t) + \mathrm{B}\mathbf{u}_R(t) + \mathrm{B}\mathbf{f}_a(t) + \mathbf{f}_c(t) \\ \mathbf{y}(t) & = & \mathrm{C}\mathbf{x}(t) + \mathbf{f}_y(t) \end{cases}$$

The general case

- State–space model

$$\begin{cases} \mathbf{x}(t+1) & = & \mathrm{A}\mathbf{x}(t) + \mathrm{B}\mathbf{u}(t) + \mathrm{L}_1\mathbf{f}(t) \\ \mathbf{y}(t) & = & \mathrm{C}\mathbf{x}(t) + \mathrm{L}_2\mathbf{f}(t) \end{cases}$$

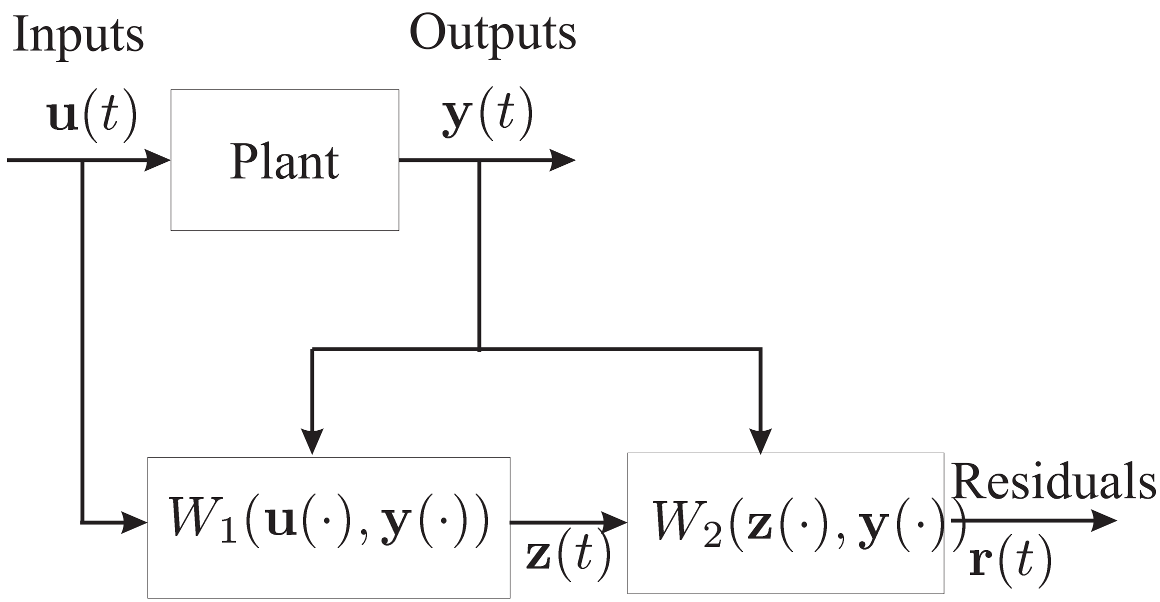
- Input–Output model

$$\mathbf{y}(s) = \mathrm{G}_{yu}(s)\mathbf{u}(s) + \mathrm{G}_{yf}(s)\mathbf{f}(s)$$

where

$$\begin{cases} \mathbf{G}_{yu}(s) & = & \mathrm{C}(s\mathrm{I} - \mathrm{A})^{-1}\mathrm{B} \\ \mathbf{G}_{yf}(s) & = & \mathrm{C}(s\mathrm{I} - \mathrm{A})^{-1}\mathrm{L}_1 + \mathrm{L}_2 \end{cases}$$

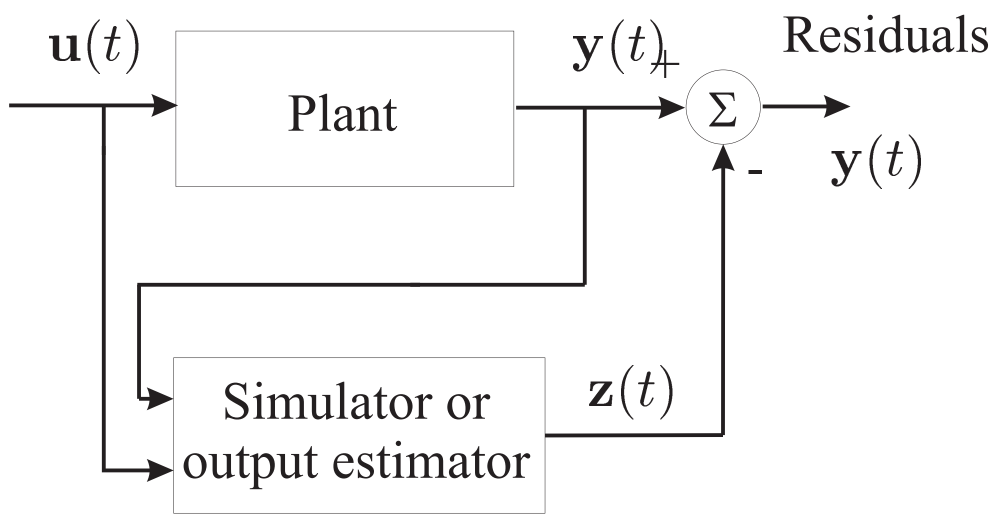
Residual Generator Structure



$$\begin{cases} \mathbf{z}(t) & = & W_1(\mathbf{u}(\cdot), \mathbf{y}(\cdot)) \\ \mathbf{r}(t) & = & W_2(\mathbf{z}(\cdot), \mathbf{y}(\cdot)) \end{cases}$$

Fault free: $\mathbf{r}(t) = \mathbf{0}$ Faulty: $\mathbf{r}(t) \neq \mathbf{0}$

Residual generator via output estimator.



$$W_1(\mathbf{u}(\cdot), \mathbf{y}(\cdot)) = \mathrm{M}\mathbf{y}(t)$$
$$W_2(\mathbf{z}(\cdot), \mathbf{y}(\cdot)) = \mathrm{W}(\mathbf{z}(t) - \mathrm{M}\mathbf{y}(t))$$

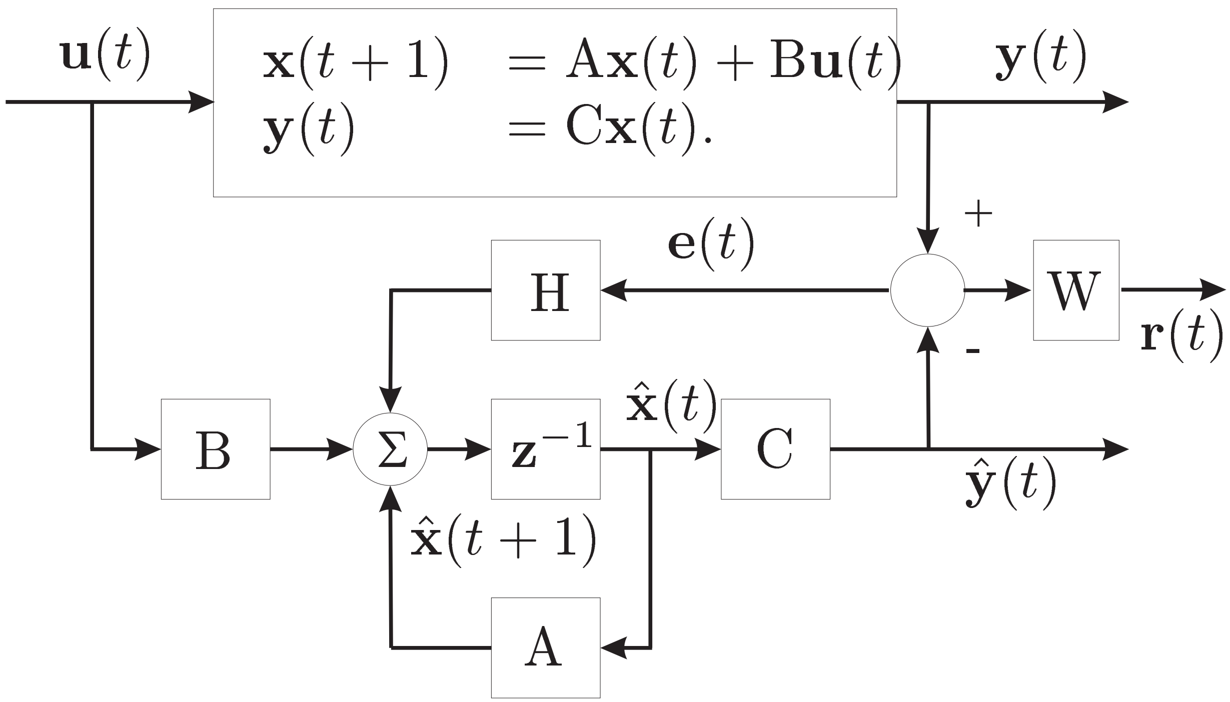
Fault detection.

- Residual limit value checking:

$$\begin{cases} \mathbf{r}(t) \leq \epsilon & \text{for} \quad \mathbf{f}(t) = \mathbf{0} \\ \mathbf{r}(t) > \epsilon & \text{for} \quad \mathbf{f}(t) \neq \mathbf{0} \end{cases}$$

- If the residual exceeds the threshold, a fault may be occurred.

Residual Generation with Dynamic Observers



Observer: $\begin{cases} \hat{\mathbf{x}}(t+1) & = & \mathrm{A}\hat{\mathbf{x}}(t) + \mathrm{B}\mathbf{u}(t) + \mathrm{H}\mathbf{e}(t) \\ \mathbf{e}(t) & = & \mathbf{y}(t) - \mathrm{C}\hat{\mathbf{x}}(t). \end{cases}$

State estimation error

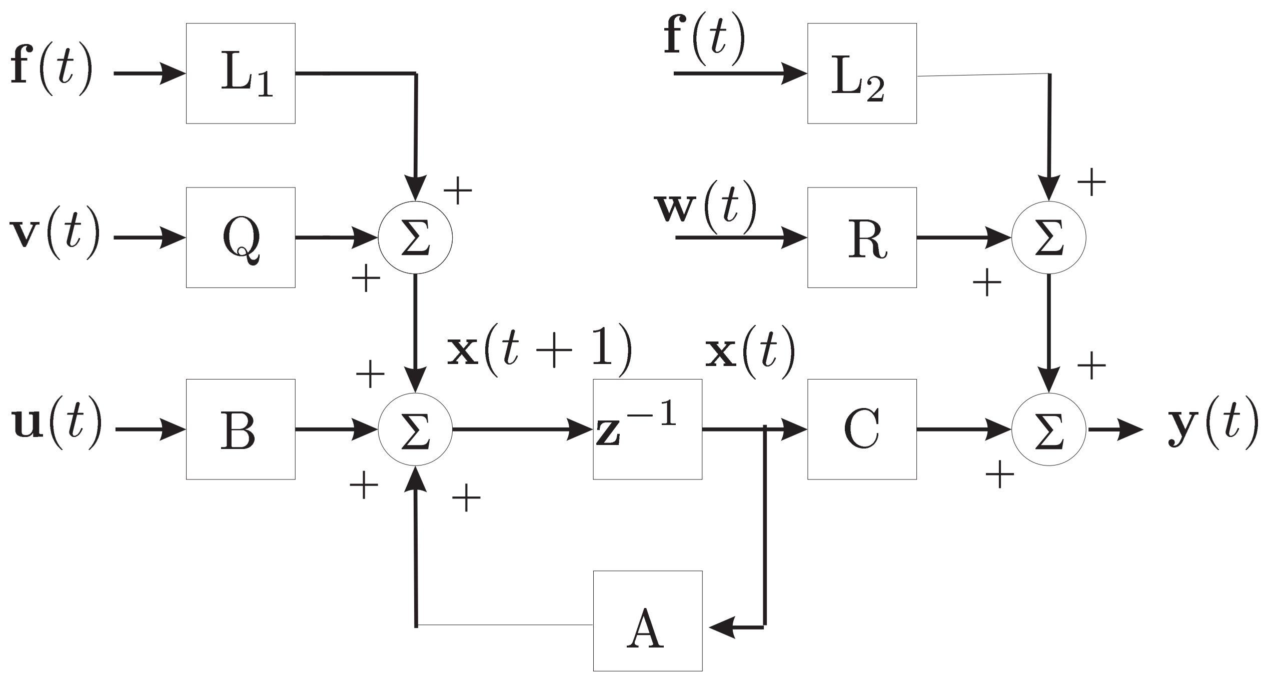
$$\begin{cases} \mathbf{e}_x(t) & = & \mathbf{x}(t) - \hat{\mathbf{x}}(t) \\ \mathbf{e}_x(t+1) & = & (\mathrm{A} - \mathrm{HC})\mathbf{e}_x(t). \end{cases}$$

The state error $\mathbf{e}_x(t)$ vanishes asymptotically

$$\lim_{t \to \infty} \mathbf{e}_x(t) = \mathbf{0}$$

if the observer is stable (by means of proper design of the observer feedback H).

Faults in the observer scheme



$$\begin{cases} \mathbf{x}(t+1) & = & \mathrm{A}\mathbf{x}(t) + \mathrm{B}\mathbf{u}(t) + \mathrm{Q}\mathbf{v}(t) + \mathrm{L}_1\mathbf{f}(t) \\ \mathbf{y}(t) & = & \mathrm{C}\mathbf{x}(t) + \mathrm{R}\mathbf{w}(t) + \mathrm{L}_2\mathbf{f}(t) \end{cases}$$

State estimation error in presence of faults - I

- Hypothesis: $\mathbf{v}(t) = \mathbf{0}$ and $\mathbf{w}(t) = \mathbf{0}$

$$\mathbf{e}_x(t+1) = (\mathrm{A} - \mathrm{HC})\mathbf{e}_x(t) + \mathrm{L}_1\mathbf{f}(t) - \mathrm{HL}_2\mathbf{f}(t)$$
$$\mathbf{e}(t) = \mathrm{C}\mathbf{e}_x(t) + \mathrm{L}_2\mathbf{f}(t)$$

- where
  - $\mathrm{C}\mathbf{e}_x(t)$ represent faults modulated by system dynamics.
  - $\mathrm{L}_2\mathbf{f}(t)$ represents fault affecting output directly.

- In fact, considering the $Z - trans.$ of $\mathbf{e}(t)$:

$$\mathbf{e}(z) = C(z\mathrm{I} - \mathrm{A} + \mathrm{HC})^{-1}(\mathrm{L}_1\mathbf{f}(z) - \mathrm{HL}_2\mathbf{f}(z)) + \mathrm{L}_2\mathbf{f}(z)$$

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State estimation error in presence of faults - II

- If faults appear as changes $\Delta\mathrm{A}$ or $\Delta\mathrm{B}$:

$$\begin{cases} \mathbf{x}(t+1) & = & (\mathrm{A}+\Delta\mathrm{A})\mathbf{x}(t) + (\mathrm{B}+\Delta\mathrm{B})\mathbf{u}(t) \\ \mathbf{y}(t) & = & \mathrm{C}\mathbf{x}(t) \end{cases}$$

while the state $\mathbf{e}_x(t)$ and the output estimation $\mathbf{e}(t)$ errors

$$\begin{cases} \mathbf{e}_x(t+1) & = & (\mathrm{A}-\mathrm{HC})\mathbf{e}_x(t) + \Delta\mathrm{A}\mathbf{x}(t) + \Delta\mathrm{B}\mathbf{u}(t) \\ \mathbf{e}(t) & = & \mathrm{C}\mathbf{e}_x(t). \end{cases}$$

- The changes $\Delta\mathrm{A}$ and $\Delta\mathrm{B}$ are then multiplicative faults.

Parity relation (vector) method

- The FDI problem can be posed either in Transfer Function and State Space forms.
- Let's consider the Transfer Function approach

$$y(z) = \mathrm{M}(z)u(z) + \mathrm{S}(z)p(z)$$

$y(t)$	system outputs
$u(t)$	known (observed) inputs
$p(t)$	(additive) unknown inputs

$u(z)$ and $p(z)$ can be further decomposed into

$$u(z)\begin{cases} u_c(z) & \text{Controlled input} \\ u_m(z) & \text{measured inputs} \end{cases}$$

$$p(z)\begin{cases} p_f(z) & \text{additive faults (input and output sensor faults, actuator faults)} \\ p_d(z) & \text{additive distrubance} \end{cases}$$

The system becomes:

$$y(z) = [\mathrm{M}_c(z)\ \mathrm{M}_m(z)] \begin{bmatrix} u_c(z) \\ u_m(z) \end{bmatrix} + [\mathrm{S}_f(z)\ \mathrm{S}_d(z)] \begin{bmatrix} p_f(z) \\ p_d(z) \end{bmatrix}$$

Parametric (system) faults and model error can be represented by

$$y(z) = \mathrm{M}(z)u(z) + \Delta\mathrm{M}(z)u(z)$$

$$\Delta\mathrm{M}(z) \begin{cases} \Delta\mathrm{M}_f(z) & \text{parametric faults} \\ \Delta\mathrm{M}_d(z) & \text{modelling errors} \end{cases}$$

leading to:

$$y(z) = \mathrm{M}(z)u(z) + [\Delta\mathrm{M}_f(z)\ \Delta\mathrm{M}_d(z)]u(z)$$

Residual generation for additive faults

- Residual generation:

$$r(z) = W(z)[y(t) - M(z)u(z)]$$

where $r(z)$ is the residual and $\mathrm{W}(z)$ is the residual generation matrix.

- substituting $y(t)$ yelds the internal form

$$r(z) = \mathrm{W}(z)\mathrm{S}(z)p(z)$$

- One may specify the desired response as

$$r(z) = \mathrm{Z}(z)p(z)$$

- comparing this to $r(z) = \mathrm{W}(z)\mathrm{S}(z)p(z)$, we obtain the implementation equation

$$\mathrm{W}(z)\mathrm{S}(z) = \mathrm{Z}(z)$$

where $\mathrm{S}(z)$ and $\mathrm{Z}(z)$ are given and $\mathrm{W}(z)$ is to be found.

The equivalence of linear residual generators.

- Any residual generator can be described as

$$r(z) = \mathrm{W}(z)y(z) + \mathrm{V}(z)u(z)$$

- $r(z)$ should be zero in absence of faults or distrubance. This requires

$$\mathrm{W}(z)\mathrm{M}(z) + \mathrm{V}(z) = 0$$

- leading to the “parity relation”:

$$r(z) = W(z)[y(t) - M(z)u(z)]$$

Robust Residual Generation Problem

- Mathematical description of the monitored system that includes any kind of modelling uncertainty:

$$\dot{x} = (A + \Delta A)x(t) + (B + \Delta B)u(t) + E_1 d(t) + L_1 f(t)$$
$$y(t) = (C + \Delta C)x(t) + E_2 d(t) + L_2 f(t)$$

- Transfer function:

$$y(s) = (G_u(s) + \Delta G_u(s))u(s) + G_d(s)d(s) + G_f(s)f(s)$$

- residual generator:

$$r(s) = H_y(s)G_f(s)f(s) + \overbrace{H_y(s)\Delta G_u(s) + H_y(s)G_d(s)d(s)}^{\text{Spurious terms}}$$

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Robustness to disturbances

- Total decoupling to disturbance:

$$H_y(s)G_d(s) = 0$$

- If only a partial decoupling can be achieved, then a performance index can be defined:

$$J = \frac{H_y(j\omega)G_d(j\omega)}{H_y(j\omega)G_f(j\omega)}$$

Adaptive threshold in robust FDI

- Residual uncertainty arising from modelling errors:

$$r(s) = H_y(s)\Delta G_u(s)$$

- Assuming that the modelling errors are bounded:

$$||\Delta G_u(s)|| \leq \delta$$

- fault free residual will be bounded as

$$\begin{aligned} ||r(j\omega)|| &= ||H_y(j\omega)\Delta G_u(j\omega)u(j\omega)|| \leq \\ &\leq ||H_y(j\omega)u(j\omega)||||\Delta G_u(j\omega)|| \leq \delta||H_y(j\omega)u(j\omega)|| \end{aligned}$$

- THerefore an adaptive threshold can be generated as

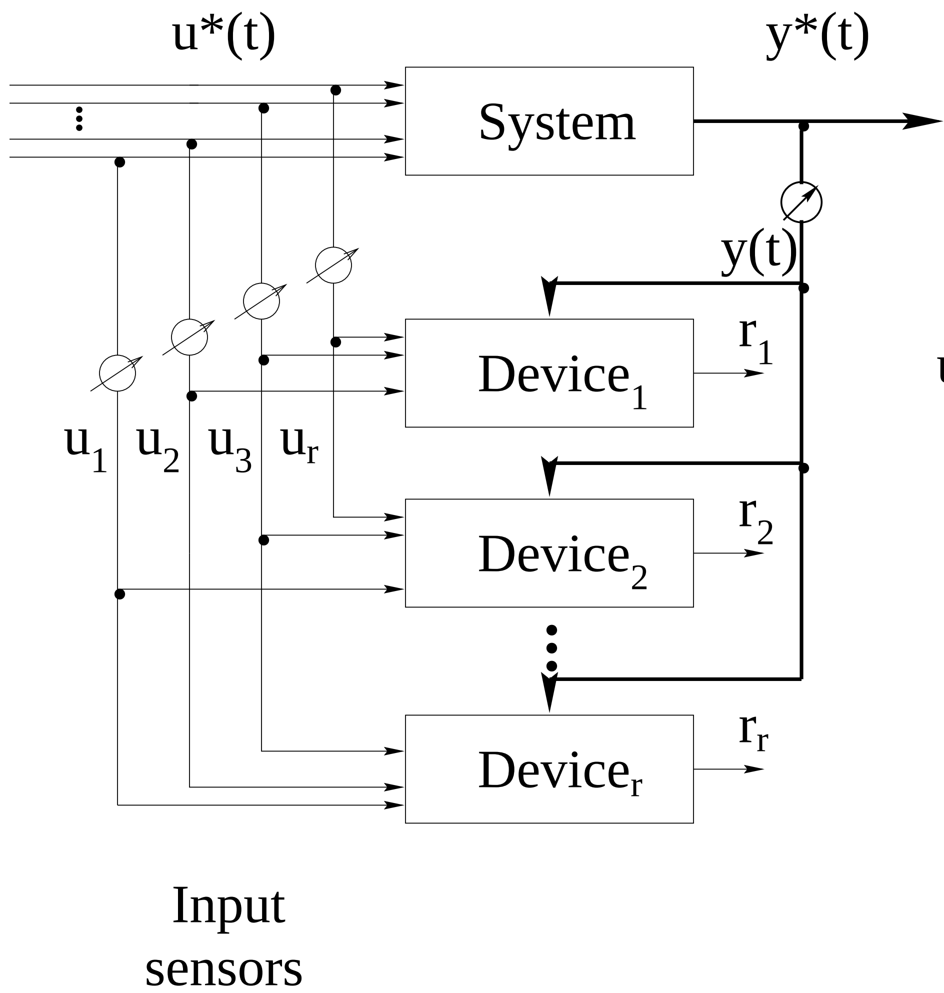
$$T(s) = \delta H_y(s)u(s)$$

- a fault is declared if:

$$||r(t)|| \leq ||T(t)||$$

Unknown Input Observer

- The i-th device is driven by all but the i–th input sensor and all outputs of the system.
- The residual function which is sensitive to all but the i–th input sensor fault.
- A fault on the i–th input sensor affects all the residual functions except that of the device which is insensitive to the i-th input.



$$u_i(t) = u_i^*(t) + \tilde{u}_i(t) + f_{u_i}(t)$$

$$y(t) = y^*(t) + \tilde{y}(t)$$

$$(i=1,\ldots,r)$$

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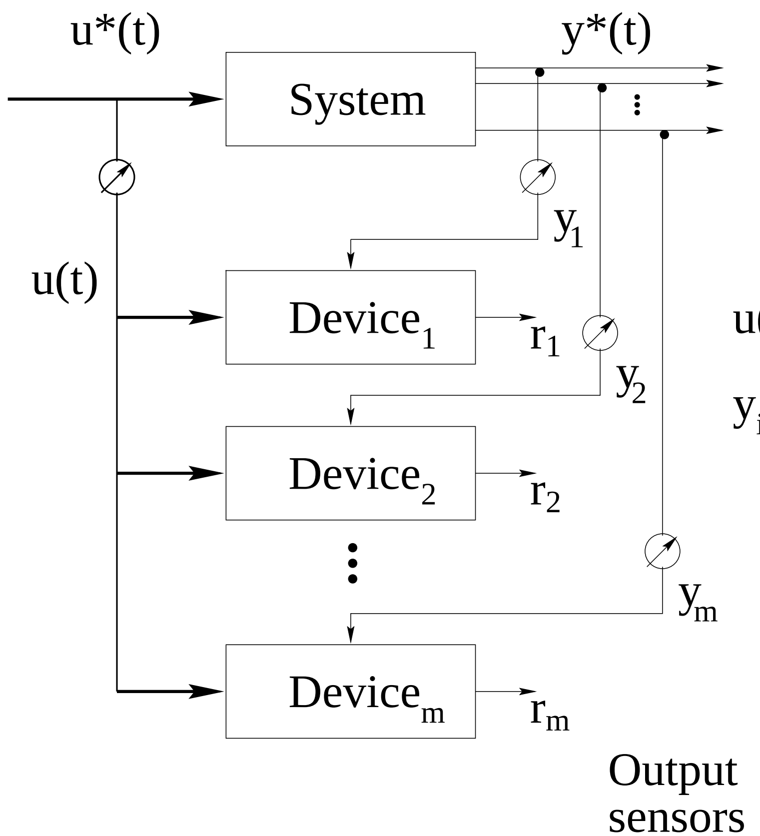
Output sensor fault diagnosis



$$u(t) = u^*(t) + \tilde{u}(t)$$

$$y_i(t) = y_i^*(t) + \tilde{y}_i(t) + f_{y_i}(t)$$

$$(i=1,\ldots,m)$$