Port-controlled Hamiltonian systems: towards a theory for control and design of nonlinear physical systems*

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Abstract

It is shown how *network modeling* of lumped-parameter physical systems naturally leads to a geometrically defined class of systems, called *port-controlled Hamiltonian systems with dissipation*. The structural properties of these systems are discussed, in particular the existence of Casimir functions and their implications for stability. It is shown how a power-conserving interconnection of port-controlled Hamiltonian systems defines another port-controlled Hamiltonian system, and how this may be used for design and for control by shaping the internal energy.
keywords: nonlinear control, modeling, passivity, Hamiltonian systems, interconnection

1 Introduction

Nonlinear systems and control theory has witnessed tremendous developments over the last three decades, see for example the textbooks [7, 17]. Especially the introduction of geometric tools like Lie brackets of vector fields on manifolds has greatly advanced the theory, and has enabled the proper generalization of many fundamental concepts known for *linear control systems* to the nonlinear world. While the emphasis in the eighties has been primarily on the *structural* analysis of smooth nonlinear dynamical control systems, in the nineties this has been combined with analytic techniques for stability, stabilization and robust control, leading e.g. to backstepping techniques and nonlinear H_∞- control. Moreover, in the last decade the theory of *passive* systems, and its implications for regulation and tracking, has undergone a remarkable revival. This last development was also spurred by work in robotics on the possibilities of *shaping* by feedback the *physical energy* in such a way that it can be used as a suitable Lyapunov function for the control purpose at hand, see e.g. the influential paper [29]. This has led to what is sometimes called *passivity-based control*, see e.g. [18]. Many other important developments have taken place, and much attention has been paid to special subclasses of systems like mechanical systems with nonholonomic constraints. All this has resulted in a very lively research in nonlinear control, with many actual and potential applications.
In the present paper we want to stress the importance of *modeling* for nonlinear control. Of course, this is common practice for working engineers, but a general theoretical framework for modeling is also of utmost importance for the development of nonlinear control theory. In this paper we limit

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ourselves to lumped-parameter physical systems, such as mechanical and electrical systems as well as *electro-mechanical* systems, although in principle the framework can be extended to *distributed-parameter* systems, which in fact is a topic of current research. We discuss how *network modeling* of such systems naturally leads to a geometrically defined class of systems called *port-controlled Hamiltonian systems with dissipation* (PCHD systems). These systems are determined by an internal interconnection structure, a Hamiltonian defined as the total stored energy, and a resistive structure. The structural properties of these systems are investigated through geometric tools stemming from the theory of Hamiltonian systems. It is indicated how the *interconnection* of PCHD systems leads to another PCHD system, and how this may be exploited for control and design. In particular, we investigate the existence of Casimir functions for the feedback interconnection of a plant PCHD system and a controller PCHD system, leading to a reduced PCHD system on invariant manifolds with *shaped* energy. We thus provide an interpretation of *passivity-based control* from an *interconnection* point of view. In a future paper we shall show how this framework can be extended to *implicit* port-controlled Hamiltonian systems, as often arising from first principles modeling.

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2 Port-controlled Hamiltonian systems

2.1 From the Euler-Lagrange and Hamiltonian equations to port-controlled Hamiltonian systems

Let us briefly recall the standard Euler-Lagrange and Hamiltonian equations of motion. The standard *Euler-Lagrange equations* are given as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}(q,\dot{q})\right) - \frac{\partial L}{\partial q}(q,\dot{q}) = \tau, \tag{1}$$

where $q = (q_1, \ldots, q_k)^T$ are generalized configuration coordinates for the system with k degrees of freedom, the Lagrangian L equals the difference $K - P$ between kinetic energy K and potential energy P, and $\tau = (\tau_1, \ldots, \tau_k)^T$ is the vector of generalized forces acting on the system. Furthermore, $\frac{\partial L}{\partial \dot{q}}$ denotes the column-vector of partial derivatives of $L(q, \dot{q})$ with respect to the generalized velocities $\dot{q}_1, \ldots, \dot{q}_k$, and similarly for $\frac{\partial L}{\partial q}$. In standard mechanical systems the kinetic energy K is of the form

$$K(q,\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} \tag{2}$$

where the $k \times k$ inertia (generalized mass) matrix $M(q)$ is symmetric and positive definite for all q. In this case the vector of generalized *momenta* $p = (p_1, \ldots, p_k)^T$, defined for any Lagrangian L as $p = \frac{\partial L}{\partial \dot{q}}$, is simply given by

$$p = M(q)\dot{q}, \tag{3}$$

and by defining the state vector $(q_1, \ldots, q_k, p_1, \ldots, p_k)^T$ the k second-order equations (1) transform into $2k$ first-order equations

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p}(q,p) \qquad (= M^{-1}(q)p) \\ \dot{p} &= -\frac{\partial H}{\partial q}(q,p) + \tau \end{aligned} \tag{4}$$

where

$$H(q,p) = \frac{1}{2} p^T M^{-1}(q) p + P(q) \quad (= \frac{1}{2} \dot{q}^T M(q) \dot{q} + P(q)) \tag{5}$$

is the total energy of the system. The equations (4) are called the *Hamiltonian equations* of motion, and H is called the *Hamiltonian.* The following *energy balance* immediately follows from (4):

$$\frac{d}{dt} H = \frac{\partial^T H}{\partial q}(q,p)\dot{q} + \frac{\partial^T H}{\partial p}(q,p)\dot{p} = \frac{\partial^T H}{\partial p}(q,p)\tau \;=\; \dot{q}^T \tau, \tag{6}$$

expressing that the increase in energy of the system is equal to the supplied work (*conservation of energy*).

If the potential energy is *bounded from below*, that is $\exists C > -\infty$ such that $P(q) \geq C$, then it follows that (4) with inputs $u = \tau$ and outputs $y = \dot{q}$ is a *passive* (in fact, *lossless*) state space system with storage function $H(q,p) - C \geq 0$ (see e.g. [30, 6, 23] for the general theory of passive and dissipative systems). Since the energy is only defined up to a constant, we may as well as take as potential energy the function $P(q) - C \geq 0$, in which case the total energy $H(q,p)$ becomes nonnegative and thus itself is the storage function.

System (4) is an example of a *Hamiltonian system* with collocated inputs and outputs, which more generally is given in the following form

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p}(q,p) \quad , \quad (q,p) = (q_1, \ldots, q_k, p_1, \ldots, p_k) \\ \dot{p} &= -\frac{\partial H}{\partial q}(q,p) + B(q)u, \qquad u \in \mathbb{R}^m, \\ y &= B^T(q)\frac{\partial H}{\partial p}(q,p) \qquad (= B^T(q)\dot{q}), \qquad y \in \mathbb{R}^m, \end{aligned} \tag{7}$$

Here $B(q)$ is the input force matrix, with $B(q)u$ denoting the generalized forces resulting from the control inputs $u \in \mathbb{R}^m$. The state space of (7) with local coordinates (q,p) is usually called the *phase space*. Normally $m < k$, in which case we speak of an *underactuated* system.

Because of the form of the output equations $y = B^T(q)\dot{q}$ we again obtain the energy balance

$$\frac{dH}{dt}(q(t), p(t)) = u^T(t) y(t) \tag{8}$$

and if H is bounded from below, any Hamiltonian system (7) is a lossless state space system. For a system-theoretic treatment of Hamiltonian systems (7), we refer to e.g. [2, 20, 21, 3, 17].

A major generalization of the class of Hamiltonian systems (7) is to consider systems which are described in local coordinates as

$$\begin{aligned} \dot{x} &= J(x)\tfrac{\partial H}{\partial x}(x) + g(x)u, \quad x \in \mathcal{X}, u \in \mathbb{R}^m \\ y &= g^T(x)\tfrac{\partial H}{\partial x}(x), \qquad y \in \mathbb{R}^m \end{aligned} \tag{9}$$

Here $J(x)$ is an $n \times n$ matrix with entries depending smoothly on x, which is assumed to be *skew-symmetric*

$$J(x) = -J^T(x), \tag{10}$$

and $x = (x_1, \ldots, x_n)$ are local coordinates for an n-dimensional state space manifold $\mathcal{X}$. Because of (10) we easily recover the energy-balance $\frac{dH}{dt}(x(t)) = u^T(t)y(t)$, showing that (9) is lossless if $H \geq 0$.

We call (9) with J satisfying (10) a *port-controlled Hamiltonian (PCH) system* with *structure matrix* $J(x)$ and *Hamiltonian* H ([12, 13]).
As an important mathematical note, we remark that in many examples the structure matrix J will satisfy the "*integrability*" conditions

$$\sum_{l=1}^{n}\left[J_{lj}(x)\frac{\partial J_{ik}}{\partial x_l}(x)+J_{li}(x)\frac{\partial J_{kj}}{\partial x_l}(x)+J_{lk}(x)\frac{\partial J_{ji}}{\partial x_l}(x)\right]=0, \quad i,\,j,k=1,\ldots,n \tag{11}$$

In this case we may find, by Darboux's theorem (see e.g. [8]) around any point x_0 where the rank of the matrix $J(x)$ is constant, local coordinates $\tilde{x}=(q,\,p,s)=(q_1,\ldots,q_k,\,p_1,\ldots,p_k,s_1,\ldots s_l)$, with $2k$ the rank of J and $n=2k+l$, such that J in these coordinates takes the form

$$J=\begin{bmatrix} 0 & I_k & 0\\ -I_k & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \tag{12}$$

The coordinates $(q,\,p,s)$ are called *canonical* coordinates, and J satisfying (10) and (11) is called a *Poisson structure matrix*. In such canonical coordinates the equations (9) are very close to the standard Hamiltonian form (7).

Example 2.1 (LCTG circuits) Consider a controlled LC-circuit consisting of two parallel inductors with magnetic energies $H_1(\varphi_1)$, $H_2(\varphi_2)$ (φ_1 and φ_2 being the magnetic flux linkages), in parallel with a capacitor with electric energy $H_3(Q)$ (Q being the charge). If the elements are linear then $H_1(\varphi_1)=\frac{1}{2L_1}\varphi_1^2$, $H_2(\varphi_2)=\frac{1}{2L_2}\varphi_2^2$ and $H_3(Q)=\frac{1}{2C}Q^2$. Furthermore let $V=u$ denote a voltage source in series with the first inductor. Using Kirchhoff's laws one immediately arrives at the dynamical equations

$$\begin{aligned}\begin{bmatrix}\dot{Q}\\ \dot{\varphi}_1\\ \dot{\varphi}_2\end{bmatrix} &= \underbrace{\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 0\\ 1 & 0 & 0\end{bmatrix}}_{J}\begin{bmatrix}\frac{\partial H}{\partial Q}\\ \frac{\partial H}{\partial \varphi_1}\\ \frac{\partial H}{\partial \varphi_2}\end{bmatrix}+\begin{bmatrix}0\\1\\0\end{bmatrix}u \\ y &= \frac{\partial H}{\partial \varphi_1} \qquad \text{(= current through first inductor)}\end{aligned} \tag{13}$$

with $H(Q,\varphi_1,\varphi_2):=H_1(\varphi_1)+H_2(\varphi_2)+H_3(Q)$ the total energy. Clearly the matrix J is skew-symmetric, and since J is constant it trivially satisfies (11). In [14] it has been shown that in this way every LC-circuit with independent elements can be modelled as a port-controlled Hamiltonian system, with the constant skew-symmetric matrix J being solely determined by the network topology (i.e., Kirchhoff's laws). Furthermore, also any LCTG-circuit with independent elements can be modelled as a PCH system, with J determined by Kirchhoff's laws *and* the constitutive relations of the transformers T and gyrators G. □

Another important class of PCH systems are mechanical systems as arising from *reduction by a symmetry group*, such as Euler's equations for a rigid body. A third important class of systems is constituted by mechanical systems with *kinematic constraints*. Consider as before a mechanical system with k degrees of freedom, locally described by k configuration variables $q=(q_1,\ldots,q_k)$. Suppose that there are constraints on the generalized velocities $\dot{q}$, described as

$$A^T(q)\dot{q}=0, \tag{14}$$

with $A(q)$ a $r \times k$ matrix of rank r everywhere (that is, there are r independent kinematic constraints). Classically, the constraints (14) are called *holonomic* if it is possible to find new configuration coordinates $\overline{q} = (\overline{q}_1, \ldots, \overline{q}_k)$ such that the constraints are equivalently expressed as

$$\dot{\overline{q}}_{k-r+1} = \dot{\overline{q}}_{n-r+2} = \cdots = \dot{\overline{q}}_k = 0 \ , \tag{15}$$

in which case one can eliminate the configuration variables $\overline{q}_{k-r+1}, \ldots, \overline{q}_k$, since the kinematic constraints (15) are equivalent to the *geometric* constraints

$$\overline{q}_{k-r+1} = c_{k-r+1}, \ldots, \overline{q}_k = c_k \ , \tag{16}$$

for certain constants $c_{k-r+1}, \ldots, c_k$ determined by the initial conditions. Then the system reduces to an *unconstrained* system in the remaining configuration coordinates $(\overline{q}_1, \ldots, \overline{q}_{k-r})$. If it is *not* possible to find coordinates $\overline{q}$ such that (15) holds (that is, if we are not able to *integrate* the kinematic constraints as above), then the constraints are called *nonholonomic.*
The equations of motion for the mechanical system with Lagrangian $L(q, \dot{q})$ and constraints (14) are given by the Euler-Lagrange equations (Neimark & Fufaev, [16])

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} &= A(q)\lambda + B(q)u, \qquad \lambda \in \mathbb{R}^r, u \in \mathbb{R}^m \\ A^T(q)\dot{q} &= 0 \end{aligned} \tag{17}$$

where $B(q)u$ are the external forces (controls) applied to the system, for some $k \times m$ matrix $B(q)$, while $A(q)\lambda$ are the *constraint forces*. Defining as before (cf. (3)) the generalized momenta the constrained Euler-Lagrange equations (17) transform into *constrained Hamiltonian equations* (compare with (7)),

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p}(q, p) \\ \dot{p} &= -\frac{\partial H}{\partial q}(q, p) + A(q)\lambda + B(q)u \\ y &= B^T(q)\frac{\partial H}{\partial p}(q, p) \\ 0 &= A^T(q)\frac{\partial H}{\partial p}(q, p) \end{aligned} \tag{18}$$

with $H(q, p) = \frac{1}{2}p^T M^{-1}(q)p + P(q)$ the total energy, and $A(q)\lambda$ the constraint forces. One way of proceeding with these equations is to *eliminate* the constraint forces, and to *reduce* the equations of motion to the constrained state space $\mathcal{X}_c = \{(q, p) \mid A^T(q)\frac{\partial H}{\partial p}(q, p) = 0\}$. In [25] it has been shown that this leads to a port-controlled Hamiltonian system (9). Furthermore, the structure matrix J_c of the resulting PCH system satisfies the integrability conditions (11) if and only if the constraints (14) are *holonomic.* (In fact, if the constraints are holonomic then the coordinates s as in (12) can be taken to be equal to the "integrated constraint functions" $\overline{q}_{k-r+1}, \ldots, \overline{q}_k$ of (16).)

2.2 Basic properties of port-controlled Hamiltonian systems

Port-controlled Hamiltonian systems (9) not only can be seen as a generalization of the classical Hamiltonian equations of motion, but they naturally arise from a *network modeling* of (complex) physical systems without dissipative elements, see our papers [12, 13, 9, 14, 26, 24, 15, 22]. Recall that a port-controlled Hamiltonian system is defined by a state space manifold $\mathcal{X}$ endowed with a *triple* (J, g, H). The pair $(J(x), g(x))$, $x \in \mathcal{X}$, captures the *interconnection structure* of the system, with $g(x)$ modeling in particular the *ports* of the system. This is very clear in Example 2.1, where the pair $(J(x), g(x))$ is determined by Kirchhoff's laws, the paradigmatic example of a power-conserving interconnection structure, but it naturally holds for other physical systems as well. Independently from the interconnection structure, the function $H : \mathcal{X} \to \mathbb{R}$ defines the total stored *energy* of the system.

PCH systems are intrinsically *modular* in the sense that a power-conserving interconnection of a number of PCH systems again defines a PCH system, with its overall interconnection structure determined by the interconnection structures of the composing individual PCH systems together with their power-conserving interconnection, and the Hamiltonian just the sum of the individual Hamiltonians (see [24, 22, 4]).

From the structure matrix $J(x)$ of a port-controlled Hamiltonian system one can directly extract useful information about the dynamical properties of the system. Since the structure matrix is directly related to the modeling of the system (capturing the interconnection structure) this information usually has a direct physical interpretation. A very important property is the possible existence of dynamical invariants *independent* of the Hamiltonian H. Consider the set of p.d.e.'s

$$\frac{\partial^T C}{\partial x}(x) J(x) = 0, \qquad x \in \mathcal{X}, \tag{19}$$

in the unknown (smooth) function $C : \mathcal{X} \to \mathbb{R}$. If (19) has a solution C then it follows that the time-derivative of C along the port-controlled Hamiltonian system (9) satisfies

$$\frac{dC}{dt} = \frac{\partial^T C}{\partial x}(x) J(x) \frac{\partial H}{\partial x}(x) + \frac{\partial^T C}{\partial x}(x) g(x) u = \frac{\partial^T C}{\partial x}(x) g(x) u \tag{20}$$

Hence, for the input $u = 0$, or for *arbitrary* input functions if additionally $\frac{\partial^T C}{\partial x}(x) g(x) = 0$, the function $C(x)$ *remains constant* along the trajectories of the port-controlled Hamiltonian system, *irrespective* of the precise form of the Hamiltonian H. A function $C : \mathcal{X} \to \mathbb{R}$ satisfying (19) is called a *Casimir function* (of the structure matrix $J(x)$).

It follows that the level sets $L_C := \{x \in \mathcal{X} | C(x) = c\}$, $c \in \mathbb{R}$, of a Casimir function C are *invariant* sets for the autonomous Hamiltonian system $\dot{x} = J(x)\frac{\partial H}{\partial x}(x)$, while the dynamics *restricted* to any level set L_C is given as the *reduced* Hamiltonian dynamics

$$\dot{x}_C = J_C(x_C) \frac{\partial H_C}{\partial x}(x_C) \tag{21}$$

with H_C and J_C the *restriction* of H, respectively J, to L_C. The existence of Casimir functions has immediate consequences for stability analysis of (9) for $u = 0$. Indeed, if $C_1, \cdots, C_r$ are Casimirs, then by (19) not only $\frac{dH}{dt} = 0$ for $u = 0$, but

$$\frac{d}{dt}\left(H + H_a(C_1, \cdots, C_r)\right)(x(t)) = 0 \tag{22}$$

for *any* function $H_a : \mathbb{R}^r \to \mathbb{R}$. Hence, if H is not positive definite at an equilibrium $x^* \in \mathcal{X}$, then $H + H_a(C_1, \cdots, C_r)$ may be rendered positive definite at x^* by a proper choice of H_a, and thus may serve as a Lyapunov function. This method for stability analysis is called the *Energy-Casimir method*, see e.g. [8].

Example 2.2 (Example 2.1 continued) The quantity $\phi_1 + \phi_2$ is a Casimir function.

2.3 Port-controlled Hamiltonian systems with dissipation

Energy-dissipation is included in the framework of port-controlled Hamiltonian systems (9) by terminating some of the ports by resistive elements. In the sequel we concentrate on PCH systems with *linear* resistive elements $u_R = -Sy_R$ for some positive semi-definite symmetric matric $S = S^T \geq 0$, where u_R and y_R are the power variables at the resistive ports . This leads to models of the form

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\,\tfrac{\partial H}{\partial x}(x) + g(x)u \\ y &= g^T(x)\tfrac{\partial H}{\partial x}(x) \end{aligned} \tag{23}$$

where $R(x)$ is a positive semi-definite symmetric matrix, depending smoothly on x. In this case the energy-balancing property (7) takes the form

$$\frac{dH}{dt}(x(t)) = u^T(t)y(t) - \frac{\partial^T H}{\partial x}(x(t))R(x(t))\frac{\partial H}{\partial x}(x(t)) \leq u^T(t)y(t). \tag{24}$$

showing passivity if the Hamiltonian H is bounded from below. We call (23) a *port-controlled Hamiltonian system with dissipation* (PCHD system). Note that in this case *two* geometric structures play a role: the internal power-conserving interconnection structure given by $J(x)$, and an additional resistive structure given by $R(x)$.

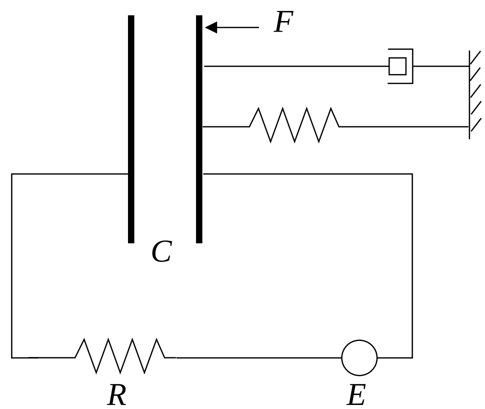

Figure 1: Capacitor microphone

Example 2.3 ([16]) Consider the capacitor microphone depicted in Figure 1. Here the capacitance $C(q)$ of the capacitor is varying as a function of the displacement q of the right plate (with mass m), which is attached to a spring (with spring constant $k > 0$) and a damper (with constant $c > 0$), and affected by a mechanical force F (air pressure arising from sound). Furthermore, E is a voltage source. The dynamical equations of motion can be written as the PCHD system

$$\begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{Q} \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & R^{-1} \end{bmatrix} \right) \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial Q} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} F + \begin{bmatrix} 0 \\ 0 \\ R^{-1} \end{bmatrix} E \tag{25}$$

$$\begin{aligned} y_1 &= \tfrac{\partial H}{\partial p} = \dot{q} \\ y_2 &= \tfrac{1}{R} \tfrac{\partial H}{\partial Q} = I \end{aligned}$$

with p the momentum, R the resistance of the resistor, I the current through the voltage source, and the Hamiltonian H being the total energy

$$H(q, p, Q) = \frac{1}{2m} p^2 + \frac{1}{2} k(q - \bar{q})^2 + \frac{1}{2C(q)} Q^2, \tag{26}$$

with $\bar{q}$ denoting the equilibrium position of the spring. Note that $F\dot{q}$ is the mechanical power, and EI the electrical power applied to the system. In the application as a microphone the voltage over the resistor will be used (after amplification) as a measure for the mechanical force F.

A rich class of examples of PCHD systems is provided by electro-mechanical systems such as induction motors, see e.g. [19]. In some examples the interconnection structure $J(x)$ is actually *varying*, depending on the mode of operation of the system, as is the case for power converters (see e.g. [5]) or for mechanical systems with variable constraints.

3 Control of port-controlled Hamiltonian systems with dissipation

The aim of this section is to discuss a general methodology for controlling PCH or PCHD systems which exploits their Hamiltonian properties in an intrinsic way. Since this exposition is based on ongoing recent research (see e.g. [10, 28, 11, 19, 23]) we only try to indicate its potential. An expected benefit of such a methodology is that it leads to physically interpretable controllers, which possess inherent robustness properties. Future research is aimed at corroborating these claims.

We have already seen that PCH or PCHD systems are *passive* if the Hamiltonian H is bounded from below. Hence in this case we can use all the results from the theory of passive systems, such as asymptotic stabilization by the insertion of *damping* by negative output feedback, see e.g. [23]. The emphasis in this section is however on the somewhat complementary aspect of *shaping the energy* of the system, which directly involves the Hamiltonian structure of the system, as opposed to the more general passivity structure.

3.1 Control by interconnection

Consider a port-controlled Hamiltonian system with dissipation (23) regarded as a plant system to be controlled. Recall the well-known result that the standard feedback interconnection of two passive systems again is a passive system; a basic fact which can be used for various stability and control purposes ([6, 18, 23]). In the same vein we consider the interconnection of the plant (23) with *another* port-controlled Hamiltonian system with dissipation

$$C: \begin{array}{l} \dot{\xi} = [J_C(\xi) - R_C(\xi)]\frac{\partial H_C}{\partial \xi}(\xi) + g_C(\xi)u_C \\ y_C = g_C^T(\xi)\frac{\partial H_C}{\partial \xi}(\xi) \end{array} \quad \xi \in \mathcal{X}_C \tag{27}$$

regarded as the *controller* system, via the standard feedback interconnection

$$\begin{aligned} u &= -y_C + e \\ u_C &= y + e_C \end{aligned} \tag{28}$$

with e, e_C external signals inserted in the feedback loop. The closed-loop system takes the form

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} &= (\underbrace{\begin{bmatrix} J(x) & -g(x)g_C^T(\xi) \\ g_C(\xi)g^T(x) & J_C(\xi) \end{bmatrix}}_{J_{cl}(x,\xi)} - \underbrace{\begin{bmatrix} R(x) & 0 \\ 0 & R_C(\xi) \end{bmatrix}}_{R_{cl}(x,\xi)}) \\ & \begin{bmatrix} \frac{\partial H}{\partial x}(x) \\ \frac{\partial H_C}{\partial \xi}(\xi) \end{bmatrix} + \begin{bmatrix} g(x) & 0 \\ 0 & g_C(\xi) \end{bmatrix} \begin{bmatrix} e \\ e_C \end{bmatrix} \\ \begin{bmatrix} y \\ y_C \end{bmatrix} &= \begin{bmatrix} g(x) & 0 \\ 0 & g_C(\xi) \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x}(x) \\ \frac{\partial H_C}{\partial \xi}(\xi) \end{bmatrix} \end{aligned} \tag{29}$$

which again is a port-controlled Hamiltonian system with dissipation, with state space given by the product space $\mathcal{X} \times \mathcal{X}_C$, total Hamiltonian $H(x) + H_C(\xi)$, inputs (e, e_C) and outputs (y, y_C). Hence the feedback interconnection of any two PCHD systems results in another PCHD system; just as in the case of passivity. This is a special case of a theorem ([23]), which says that any regular power-conserving interconnection of PCHD systems defines another PCHD system.

It is of interest to investigate the *Casimir functions* of the closed-loop system, especially those relating the state variables ξ of the controller system to the state variables x of the plant system. Indeed, from a control point of view the Hamiltonian H is *given* while H_C can be *assigned.* Thus if we can find Casimir functions $C_i(\xi, x)$, $i = 1, \cdots, r$, relating ξ to x then by the Energy-Casimir method the Hamiltonian $H + H_C$ of the closed-loop system may be replaced by the Hamiltonian $H + H_C + H_a(C_1, \cdots, C_r)$, thus creating the possibility of obtaining a suitable Lyapunov function for the closed-loop system.

Example 3.1 [27] Consider the "plant" system

$$\begin{aligned} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} \end{aligned} \tag{30}$$

with q the position and p being the momentum of the mass m, in feedback interconnection $(u = -y_C + e, u_C = y)$ with the controller system (see Figure 2)

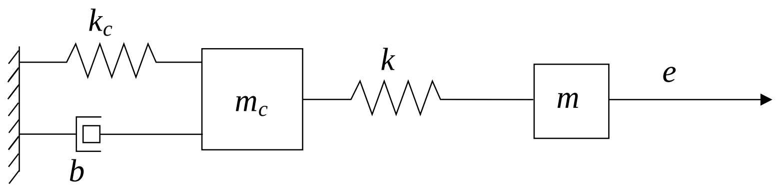

Figure 2: Controlled mass

$$\begin{bmatrix} \dot{\Delta q_c} \\ \dot{p}_c \\ \dot{\Delta q} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -b & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H_C}{\partial \Delta q_c} \\ \\ \frac{\partial H_C}{\partial p_c} \\ \\ \frac{\partial H_C}{\partial \Delta q} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_C \tag{31}$$

$$y_C = \frac{\partial H_C}{\partial \Delta q}$$

where Δq_c is the displacement of the spring k_c, Δq is the displacement of the spring k, and p_c is the momentum of the mass m_c. The plant Hamiltonian is $H(p) = \frac{1}{2m}p^2$, and the controller Hamiltonian is given as $H_C(\Delta q_c, p_c, \Delta q) = \frac{1}{2}(\frac{p_c^2}{m_c} + k(\Delta q)^2 + k_c(\Delta q_c)^2)$. The variable $b > 0$ is the damping constant, and e is an external force. The closed-loop system possesses the Casimir function

$$C(q, \Delta q_c, \Delta q) = \Delta q - (q - \Delta q_c), \tag{32}$$

implying that along the solutions of the closed-loop system

$$\Delta q = q - \Delta q_c + c \tag{33}$$

with c a constant depending on the initial conditions. With the help of LaSalle's Invariance principle it can be shown that *restricted* to the invariant manifolds (33) the system is asymptotically stable for the equilibria $q = \Delta q_c = p = p_c = 0$. □

As a special case (see [23] for amore general discussion) let us consider Casimir functions for (29) of the form

$$\xi_i - G_i(x) \qquad , \quad i = 1, \ldots, \dim \mathcal{X}_C = n_C \tag{34}$$

That means that we are looking for solutions of the p.d.e.'s (with e_i denoting the i-th basis vector)

$$\begin{bmatrix} -\frac{\partial^T G_i}{\partial x}(x) & e_i^T \end{bmatrix} \begin{bmatrix} J(x) - R(x) & -g(x)g_C^T(\xi) \\ \\ g_C(\xi)g^T(x) & J_C(\xi) - R_C(\xi) \end{bmatrix} = 0 \tag{35}$$

for $i = 1, \ldots, n_C$, relating *all* the controller state variables $\xi_1, \ldots, \xi_{n_C}$ to the plant state variables x. Denoting $G = (G_1, \ldots, G_{n_C})^T$ this means ([23]) that G should satisfy

$$\begin{aligned} &\tfrac{\partial^T G}{\partial x}(x)\, J(x) \tfrac{\partial G}{\partial x}(x) = J_C(\xi) \\ &R(x) \tfrac{\partial G}{\partial x}(x) = 0 = R_C(\xi) \\ &\tfrac{\partial^T G}{\partial x}(x)\, J(x) = g_C(\xi)\, g^T(x) \end{aligned} \tag{36}$$

In this case the reduced dynamics on any multi-level set

$$L_C = \{(x,\xi) | \xi_i = G_i(x) + c_i, i = 1, \ldots n_C\} \tag{37}$$

can be immediately recognized ([23]) as the PCHD system

$$\dot{x} = [J(x) - R(x)] \frac{\partial H_s}{\partial x}(x), \tag{38}$$

with the same interconnection and dissipation structure as before, but with *shaped* Hamiltonian H_s given by

$$H_s(x) = H(x) + H_C(G(x) + c). \tag{39}$$

In the context of actuated mechanical systems this amounts to the shaping of the *potential energy* as in the classical paper [29], see [23].

A direct interpretation of the shaped Hamiltonian H_s in terms of *energy-balancing* is obtained as follows. Since $R_C(\xi) = 0$ by (36) the controller Hamiltonian H_C satisfies $\frac{dH_C}{dt} = u_C^T y_C$. Hence along any multi-level set L_C given by (37) $\frac{dH_s}{dt} = \frac{dH}{dt} + \frac{dH_C}{dt} = \frac{dH}{dt} - u^T y$, since $u = -y_C$ and $u_C = y$. Therefore, up to a constant,

$$H_s(x(t)) = H(x(t)) - \int_0^t u^T(\tau) y(\tau) d\tau, \tag{40}$$

and the shaped Hamiltonian H_s is the original Hamiltonian H *minus the energy* supplied to the plant system (23) by the controller system (27). From a stability analysis point of view (40) can be regarded as an effective way of *generating* candidate Lyapunov functions H_s from the Hamiltonian H.

3.2 Passivity-based control of port-controlled Hamiltonian systems with dissipation

In the previous section we have seen how under certain conditions the feedback interconnection of a PCHD system having Hamiltonian H (the "plant") with another PCHD system with Hamiltonian H_C (the "controller") leads to a reduced dynamics given by (38) for the shaped Hamiltonian H_s. From a *state feedback* point of view the dynamics (38) could have been directly obtained by a state feedback $u = \alpha(x)$ such that

$$g(x)\alpha(x) = [J(x) - R(x)] \frac{\partial H_C(G(x) + c)}{\partial x} \tag{41}$$

Indeed, such an $\alpha(x)$ is given in explicit form as

$$\alpha(x) = -g_C^T(G(x) + c) \frac{\partial H_C}{\partial \xi}(G(x) + c) \tag{42}$$

The state feedback $u = \alpha(x)$ is customarily called a *passivity-based control law*, since it is based on the passivity properties of the original plant system (23) and transforms (23) into *another* passive system with *shaped* storage function (in this case H_s).

Seen from this perspective we have shown in the previous section that the passivity-based state feedback $u = \alpha(x)$ satisfying (41) *can be derived* from the interconnection of the PCHD plant system (23) with a PCHD controller system (27). This fact has some favorable consequences. Indeed, it implies that the passivity-based control law defined by (41) can be equivalently *generated* as the feedback interconnection of the passive system (23) with another passive system (27). In particular, this implies an inherent *invariance* property of the controlled system: the plant system (23), the controller system (39), as well as any other passive system interconnected to (23) in a power-conserving fashion, may change in any way as long as they remain passive, and for any perturbation of this kind the controlled system will remain stable. For a further discussion of passivity-based control from this point of view we refer to [19].

4 Conclusions and future research

Clearly, the theory presented in this paper opens up the way for many control and design problems. Its potential for set-point regulation has already received some attention (see [10, 11, 19, 23]), while the extension to *tracking problems* is wide open. In this context we also like to refer to some recent work concerned with the shaping of the *Lagrangian*, see e.g. [1]. Also the connection with multi-modal (*hybrid*) systems, corresponding to PCHD systems with varying interconnection structure, needs further investigations. Finally, our current research is concerned with the formulation of *distributed parameter systems* as port-controlled Hamiltonian systems.

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