AUTOMATIC CONTROL AND SYSTEM THEORY

FEEDBACK CONTROL 2

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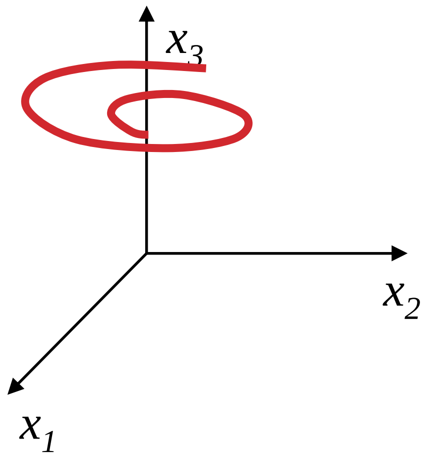
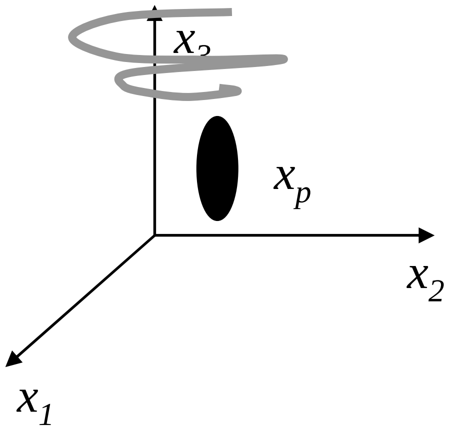
Feedback Control – non-null setpoint

- In the feedback control scheme considered so far, the origin of both the state and the input spaces has been implicitly considered as the desired equilibrium point *($x = 0$; $u = 0$).*

- In general, a final state (and input) value x_p (u_p) different from zero could be desired





Feedback Control – non-null setpoint

- Given the following asymptotically stable system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \qquad x(t_0) = x_0 \qquad x \in \mathbb{R}^{n\times 1}, \qquad u \in \mathbb{R}^{r\times 1}, \\ y(t) &= Cx(t) \qquad y \in \mathbb{R}^{m\times 1}, \end{aligned}$$

define u_p, y_p such that

x_p = set point
→ ***x* constant**
→ **vel = 0!**

$$\begin{aligned} 0 &= Ax_p + Bu_p, \\ y_p &= Cx_p \end{aligned}$$

NOTE: If $x_p \neq 0$, then $u_p \neq 0$!

- By posing

$$x_s(t) = x(t) - x_p, \qquad y_s(t) = y(t) - y_p, \qquad u_s(t) = u(t) - u_p,$$

→

$$\begin{aligned} \dot{x}_s(t) &= Ax_s(t) + Bu_s(t) \\ y_s(t) &= Cx_s(t) \end{aligned}$$

→

$$u_s(t) = K\,x_s(t),$$

Feedback Control – non-null setpoint

- From

$$u_s(t) = u(t) - u_p,$$

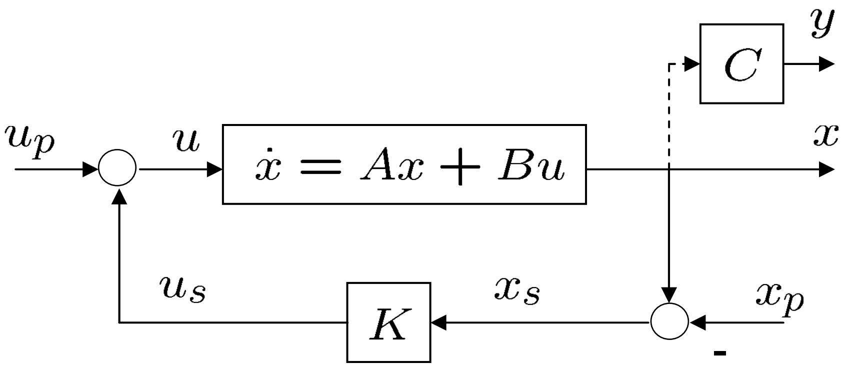
It follows

$$\begin{aligned} u(t) &= u_s(t) + u_p = Kx_s(t) + u_p = K[x(t) - x_p] + u_p \\ &= Kx(t) + u_{ps}, \qquad u_{ps} = u_p - Kx_p \end{aligned}$$

$$\dot{x}(t) = (A + BK)x(t) + Bu_{ps}$$

Feedback Control – non-null setpoint

- From $\dot{x}(t) = (A+BK)x(t) + Bu_{ps}$ the following scheme is obtained



- Since the system is asymptotically stable, the final state value is $\lim_{t\to\infty} \dot{x}(t) = 0$

$$\lim_{t\to\infty} x(t) = -(A+BK)^{-1}Bu_{ps} = -(A+BK)^{-1}(Bu_p - BKx_p)$$

$$0 = Ax_p + Bu_p \quad \to \quad Bu_p = -Ax_p$$

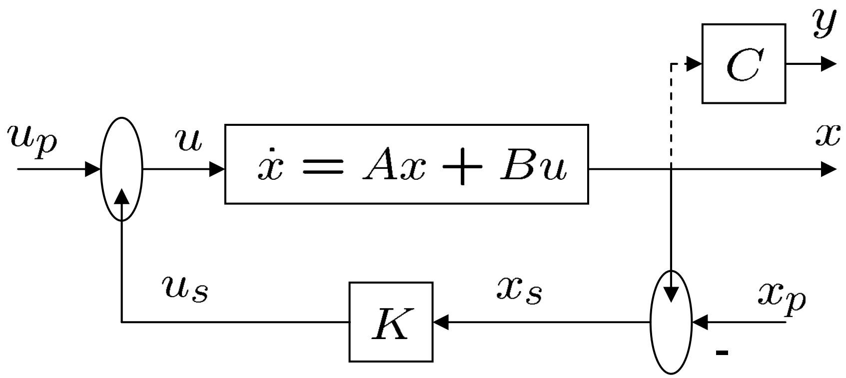
$$= (A+BK)^{-1}(A+BK)x_p = x_p$$

$$\lim_{t\to\infty} y(t) = y_p$$

$$\lim_{t\to\infty} u(t) = u_p$$

Feedback Control – non-null setpoint

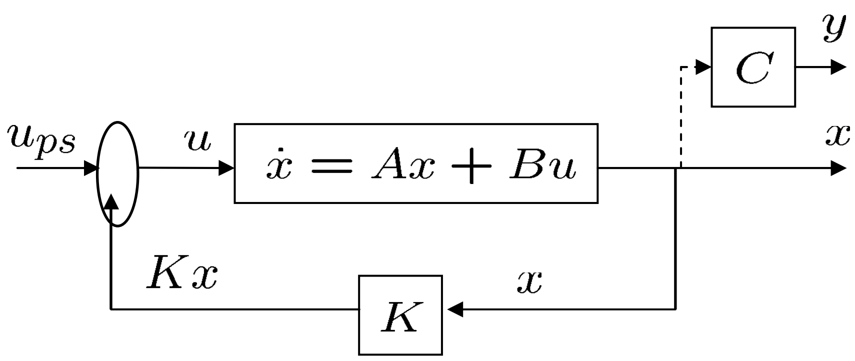
- From $\dot{x}(t) = (A+BK)x(t) + Bu_{ps}$ the following scheme is obtained



- In general, x_p is not defined but only y_p. Then, the input value must be computed by

$$u(t) = Kx(t) + u_{ps}$$

where u_{ps} is defined on the base of y_p



Feedback Control – non-null setpoint

- We need to verify if the value of u_{ps} corresponding to a given y_p can be defined.

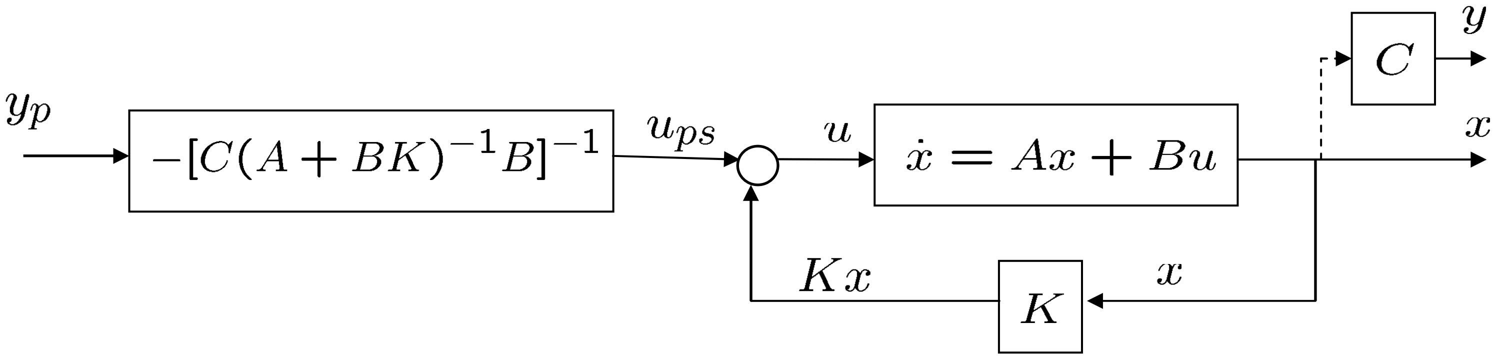
$$\begin{aligned} 0 &= Ax_p + Bu_p, \\ y_p &= Cx_p \end{aligned} \qquad \begin{aligned} u(t) &= Kx(t) + u_{ps}, \\ u_{ps} &= u_p - Kx_p \end{aligned}$$

There are three possbilities:

- ***m = r***. *The system possesses the same number of outputs (m) and inputs (r).* From

$$(A+BK)x_p + Bu_{ps} = 0 \longrightarrow u_{ps} = -[C(A+BK)^{-1}B]^{-1}y_p$$

In this case, the system is named *servomechanism*, and the output vector *y* asymptotically follows the value of the reference y_p



Feedback Control – non-null setpoint

- ***m > r***. *More outputs (m) than inputs (r).*

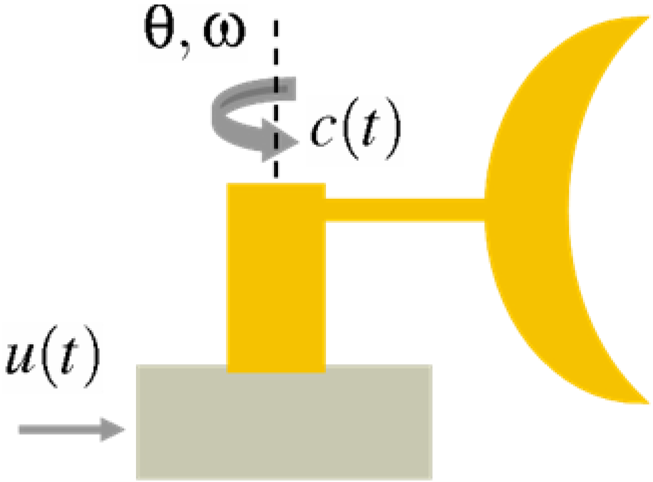
  It is possible to determine u_{ps} just for some value of the y_p, then in general the problem has no solution.

- ***m < r***. *More inputs (r) than outputs (m).*

  Several values of u_{ps} can be determined for each y_p. In this case it is preferable to increase the number of output information y(t) (increasing m) or to reduce the number of control inputs (reducing r)

Feedback Control – non-null setpoint

- ***Example:*** *Controlling the angular position of an antenna*



$$I = 1, \quad k = 1, \quad b = 2$$

$$\begin{aligned} I\,\ddot{\theta}(t) + b\,\dot{\theta}(t) &= c(t) \\ c(t) &= k\,u(t) \end{aligned}$$

$$\theta \rightarrow \theta_d$$

$$x_1 = \theta, \quad x_2 = \dot{\theta}$$

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{b}{I}x_2 + \frac{k}{I}u \end{aligned} \qquad \begin{aligned} \dot{x} &= A\,x + B\,u \\ y &= C\,x \end{aligned}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

By substituting:

$$\begin{aligned} x_{1s} &= x_1 - x_{1p} \\ x_{2s} &= x_2 \qquad (\dot{x}_{1p} = 0) \\ y_p &= x_{1p} \\ u_p &= 0 \end{aligned} \quad \longrightarrow \quad u_s = u$$

$$\longrightarrow$$

$$\begin{bmatrix} \dot{x}_{1s} \\ \dot{x}_{2s} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_{1s} \\ x_{2s} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

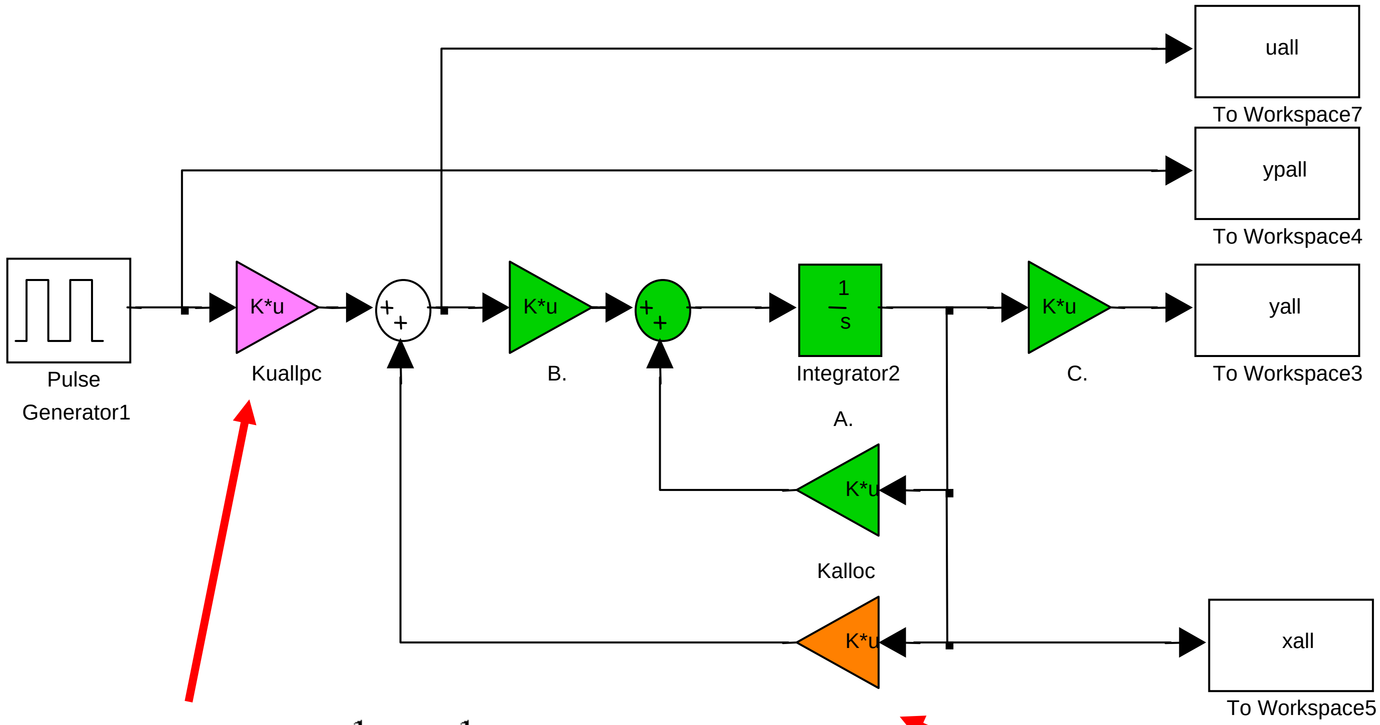
$$y_s = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{1s} \\ x_{2s} \end{bmatrix}$$

Feedback Control – non-null setpoint

- ***Example:*** *Controlling the angular position of an antenna*

The controller is designed by eigenvalues assignement

$$\lambda_1 = \lambda_2 = -3 \quad \rightarrow \quad K_a = \begin{bmatrix} -9 & -4 \end{bmatrix}$$



$$u_{ps} = -[C(A+BK_a)^{-1}B]^{-1}y_p$$

$$K_a = \begin{bmatrix} -9 & -4 \end{bmatrix}$$

Feedback Control – non-null setpoint

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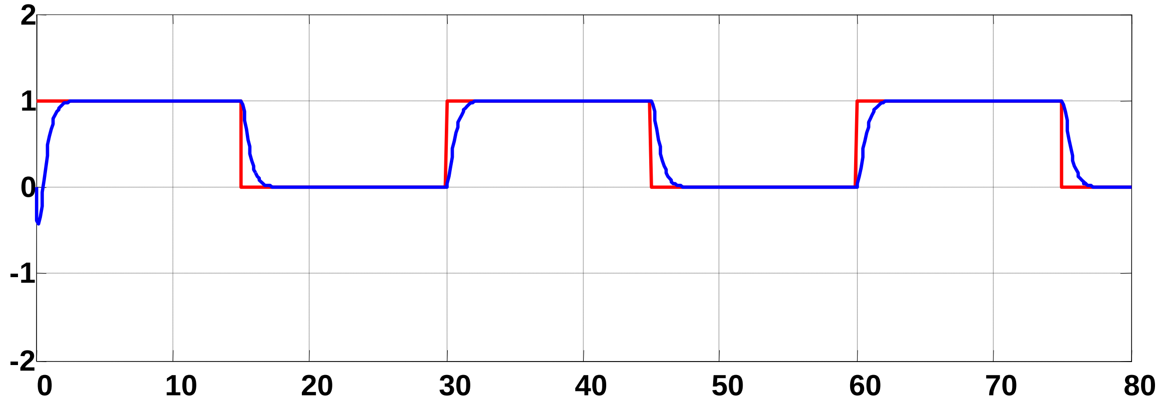
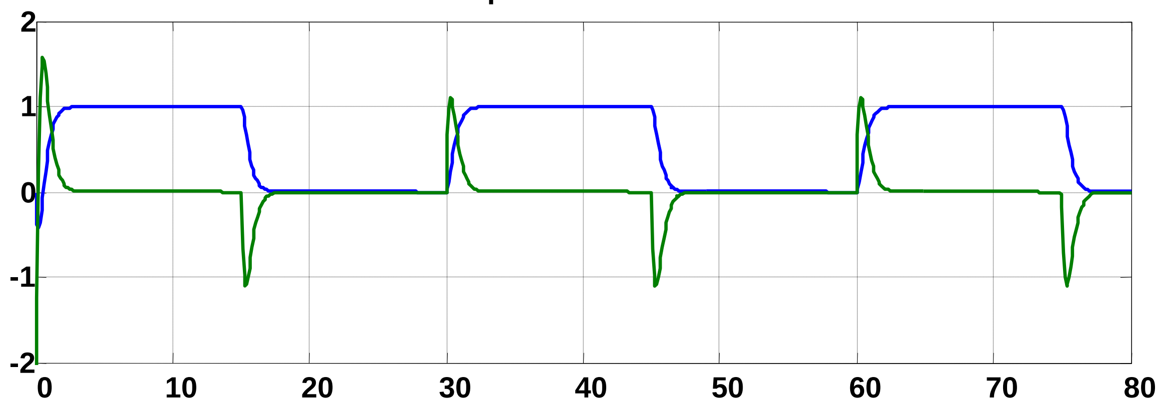
Response of yp and y



Response of x1 and x2

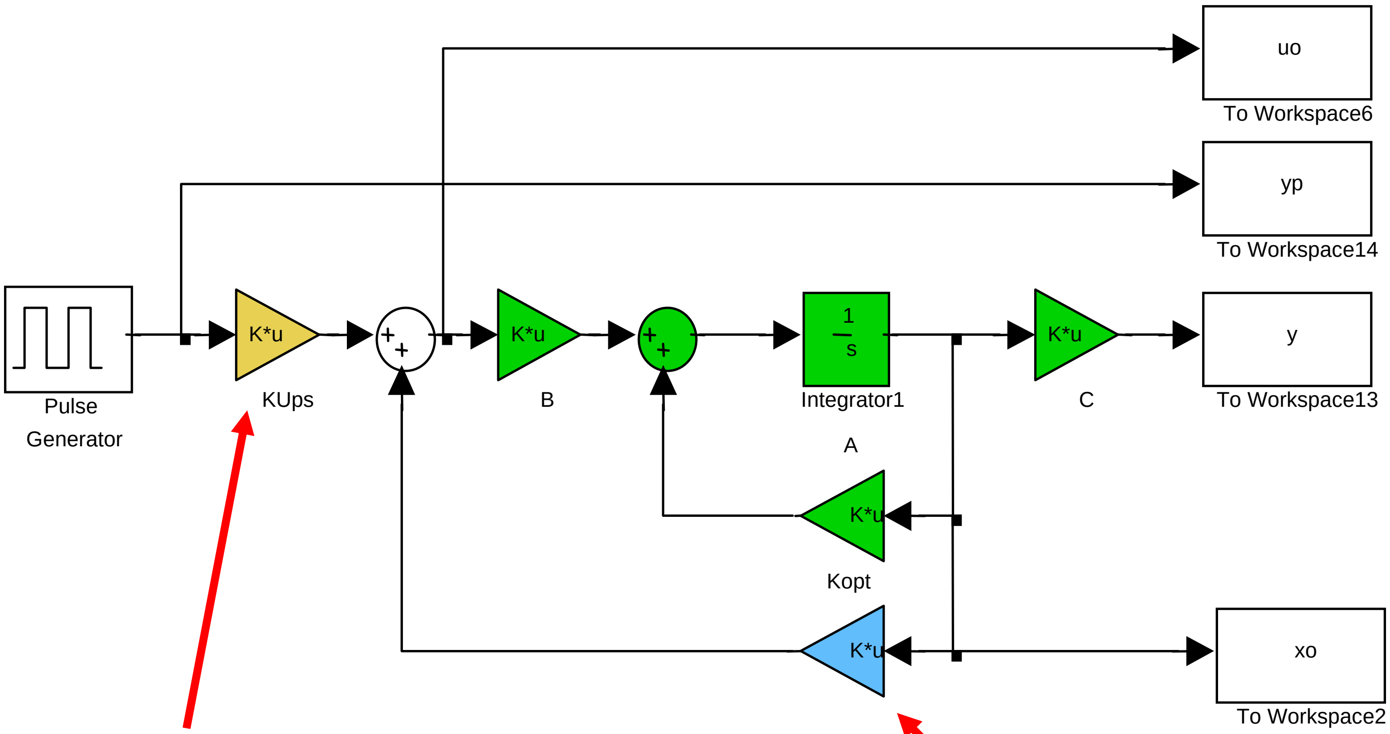


Feedback Control – non-null setpoint

- ***Example:*** *Controlling the angular position of an antenna*

With a different feedback matrix

$\lambda_1 = -0.51764, \ \lambda_2 = -1.9319$



$$u_{ps} = -[C(A+BK)^{-1}B]^{-1}y_p$$

$$K = -\begin{bmatrix} 1.0000 & 0.4495 \end{bmatrix}$$

Feedback Control – non-null setpoint

- ***Example:*** *Controlling the angular position of an antenna*

λ_1 = -0.51764, λ_2 = -1.9319

$$K = -\begin{bmatrix} 1.0000 & 0.4495 \end{bmatrix}$$