# Trajectory Planning for Robot Manipulators
# Part 3

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# Introduction

*Vibrations* are undesired phenomena often present in automatic machines. They are basically due to the presence of structural elasticity in the mechanical system, and may be generated during the normal working cycle of the machine due to several reasons.

In particular, *vibrations may be produced if trajectories with a discontinuous acceleration profile are imposed to the actuation system.*

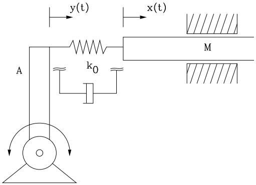
$\Longrightarrow$ Acceleration discontinuities $\rightarrow$ sudden variation of the inertial forces applied to the system.

$\Longrightarrow$ Relevant discontinuities of such forces, applied to an elastic system (i.e. any mechanical device), generate vibrational effects.

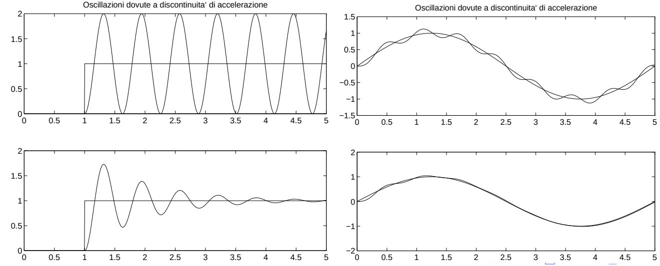
$\Longrightarrow$ Since every mechanism is characterized by some elasticity, this type of phenomenon must **always** be considered in the design of a trajectory, that therefore should have *a smooth acceleration profile or, more in general, a limited bandwidth*.

# Example

Let consider a 1-dof mechanical system (output: position $x(t)$):

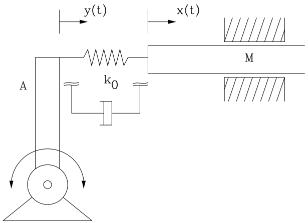



Acceleration $\ddot{x}(t)$ of the system when the (acceleration of the) input is a step or a sinusoidal function: without damping (up) and with damping (bottom).



# Models for Analysis of Vibrations

Analysis of the vibration effects $\Longrightarrow$ Models that consider the elastic, inertial and dissipative properties of the elements of the mechanical system.




The complexity level of the model is usually chosen as a compromise between the desired precision and the computational burden.

The simplest criterion is to describe the mechanical devices, that are intrinsically distributed parameter systems, as lumped parameter systems, i.e. as rigid masses (without elasticity) and elastic elements (without mass).

Energy dissipative elements are introduced in order to consider frictional phenomena among moving parts.

The numerical values of the elements that describe inertia, elasticity and dissipative effects have to be determined by energetic considerations, i.e. trying to maintain the equivalence of the kinetic and elastic energy of the model with the energy of the corresponding parts of the mechanism under study.

The description of these phenomena can be either linear or nonlinear.

# Linear model with one degree of freedom

## Some considerations

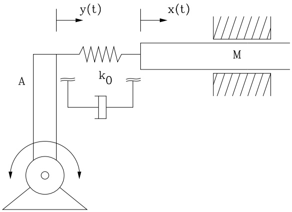
If $x(t)$ and $y(t)$ are the positions of mass M and A respectively, and $z(t) = y(t) - x(t)$, the dynamics of the system is described by:

$$m\ddot{x} + k_0(x - y) = 0$$

from which

$$\ddot{z} + \omega_0^2 z = \ddot{y}, \qquad \omega_0 = \sqrt{\frac{k_0}{m}}$$

being $\omega_0$ the natural frequency of the mechanical system.



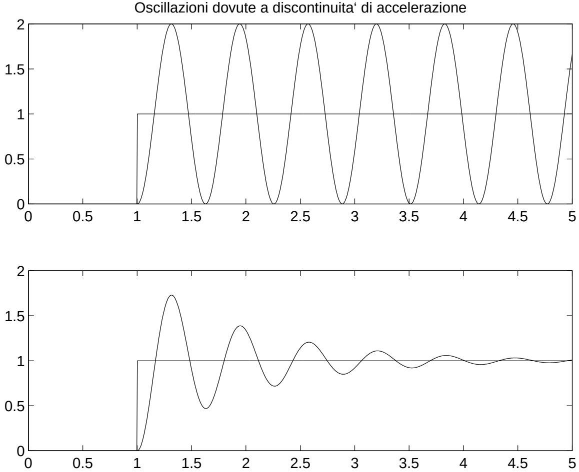

A model with viscous friction (coefficient $b$) on the mass M is described as

$$m\ddot{x} + b\dot{x} + k_0(x - y) = 0$$

# Linear model with one degree of freedom

Output of the two models with a step input

Acceleration step
($b = 0$;   $b \neq 0$)



# Linear model with one degree of freedom

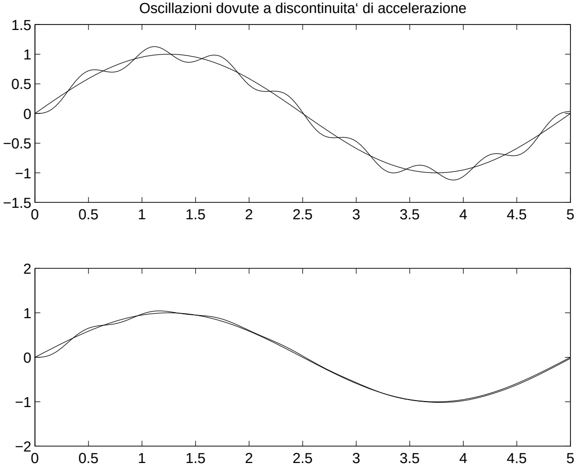
In the first case, due to oscillations the maximum value of $\ddot{x}$ is twice the value of $\ddot{y}$. Notice that this result does not depend on the stiffness $k_0$ of the mechanism:

- if $k_0$ increases, then the natural frequency $\omega_0$ increases as well, while the amplitude of $\ddot{x}$ remains constant;
- the difference $z(t)$ between the positions of M and A depends on $k_0$ (if $k_0$ increases, then, $z(t)$ decreases).

# Linear model with one degree of freedom

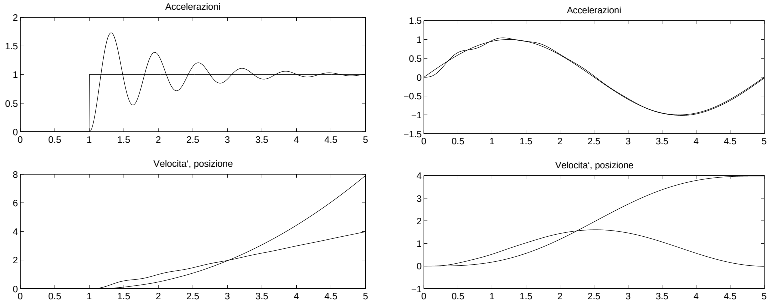
Output of the two models with a sinusoidal input:

Sinusoidal acceleration
$(b = 0; \quad b \neq 0)$



# Linear model with one degree of freedom

Although the acceleration oscillations have not a real influence on the position of the mass M, they generate a structural stress to the mechanical device.



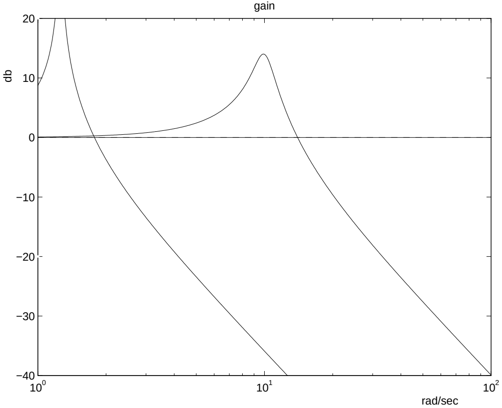

This phenomenon may be characterized by an analysis of the frequency content of the acceleration signal given as input to the system: the frequency range of the acceleration signal should be compared with the Bode diagram of the mechanism, and in particular with its natural frequency $\omega_0$.

# Linear model with one degree of freedom

Bode diagrams of the mechanical system ($b = 2, m = 1, k_0 = 100$) and of two acceleration signals:

$\rightarrow$ for the step, the Bode diagram is equal to 1 $\forall\ \omega$,

$\rightarrow$ the frequency of the sinusoidal acceleration is $\omega = 2\pi T, \quad T = 5\ s$.



# Comparison of the ‘trapezoidal’ and ‘double S’ trajectories

The ‘trapezoidal’ and ‘double S’ trajectories are very common in industrial practice, and therefore it is of interest a comparison of their main features. The criteria for the comparison are:

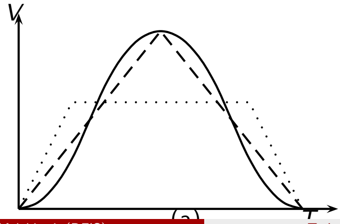
a) actuator usage;

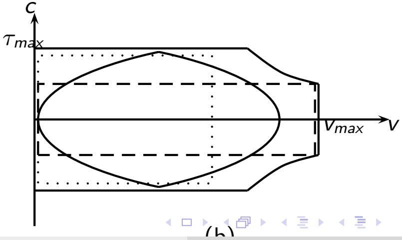
b) duration of the trajectory;

c) analysis of the frequency content (vibrations induced on the mechanical structure).

The trapezoidal, double S and triangular (limit case of trapezoidal) trajectories are considered for the analysis.

The duration of the trajectory is $T$, and the acceleration of the ‘double S’ and of the trapezoidal trajectories are $\sqrt{3}$ times the acceleration of the triangular profile, while the velocity of the trapezoidal trajectory has been set in order to obtain the same duration $T$.

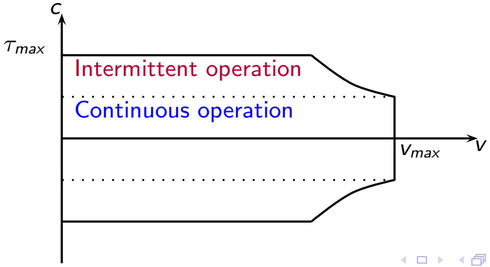


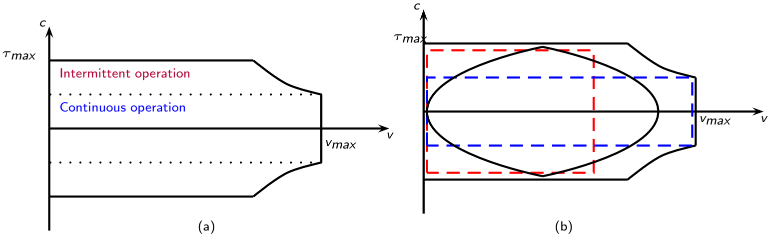
# a - Actuator usage

From the data sheet, the following characteristics (among others) of an electric drive can be obtained:

- *Continuous torque* ($\tau_c$) (or *rated torque*): torque that the motor can produce continuously without exceeding thermal limits.
- *Peak torque* ($\tau_p$): maximum torque that the motor can generate for short periods.
- *Rated speed* ($\omega_n$): maximum value of the speed at rated torque (and at rated voltage).
- *Maximum power*: maximum amount of output power generated by the motor.



# a - Actuator usage




If the motor is in the continuous operation region, it may work for an indefinite period of time, while in the intermittent region it may work only for a limited amount of time.

This limited period depends on the thermal dissipation properties of the motor and of the drive. On the other hand, different trajectories imply different utilizations of the motor, in particular with respect to the intermittent/continuous regions. As a matter of fact, the *double S trajectories allow to use the motor exploiting also the intermittent region*.

Notice that with double S trajectories the maximum torque and the maximum velocities are obtained in different time instants.

Theoretically, it is possible to enter the intermittent region also with the trapezoidal profile, but in this case the thermal power to be dissipated is higher than with double S trajectories.

# a - Actuator usage

In comparing trajectories, there are some constitutive constraints for the motors that have to be considered:

1. the requested torque cannot in any case exceed the peak torque;
2. the RMS torque of the trajectory must be not higher than the continuous torque $\tau_{cont}$.

The first is a mechanical constraint, since for all the motors there is a limit to the torque that can be generated.
The second is related to the thermal energy dissipation capability of the inverter. The trajectory must avoid heating of the motor.

If $\tau_{ss}$ and $\tau_{tr}$ are the maximum torque values of the double S and triangular trajectories, the RMS torque is computed as

$$\tau_{rms} = \sqrt{\frac{1}{T}\int_0^T C^2(t)\ dt}$$

# a - Actuator usage

With the triangular profile, one gets

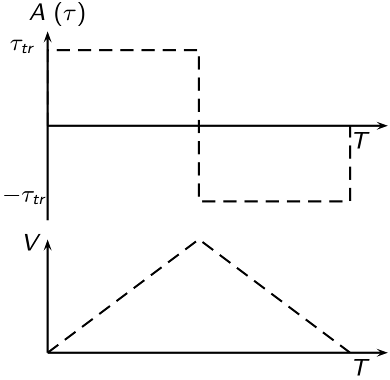
$$\tau^2(t) = \tau_{tr}^2, \qquad t \in [0, T]$$

then

$$\tau_{rms,tr} = \tau_{tr}$$

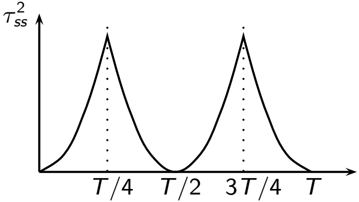
From the second constraint, one gets

$$\tau_{tr} = \tau_{cont}$$



# a - Actuator usage

With the double S trajectory (consider the case without linear velocity segments), the torque is a linear function and then $\tau_{ss}^2$ has a parabolic profile.




In this case

$$\tau_{rms,ss} = \frac{\tau_{ss}}{\sqrt{3}}$$

Therefore, the maximum torque achievable with this profile is

$$\tau_{ss} = \sqrt{3}\ \tau_{cont}$$

Notice that this trajectory allows a better exploitation of the motor with respect to the triangular profile: the maximum torque that it is possible to generate is higher $\tau_{ss} > \tau_{tr}$.

# b - Duration of the trajectory

It is simple to show that the durations $T_{tr}$ and $T_{ss}$ of the triangular and double S (without linear velocity segments) trajectories are:

$$T_{tr} = 2\sqrt{\frac{L}{A_{max}}} \qquad\qquad T_{ss} = 2\sqrt{2}\sqrt{\frac{L}{A_{max}}}$$

Therefore

$$T_{ss} = \sqrt{2}\,T_{tr}$$

As expected, being smoother, the duration of the double S trajectory is 1.41 times higher than the duration of the triangular one.

# b - Duration of the trajectory

On the other hand, with the double S trajectory it is possible to apply higher acceleration (torque) values (better usage of the actuator):

$$A_{max,ss} = \sqrt{3}\, A_{max,tr} = 1.7321\, A_{max,tr}$$

As a consequence, the duration $T_{ss}$ of the double trajectory is reduced, and therefore

$$\frac{T_{ss}}{T_{tr}} = \sqrt{2}\,\sqrt{\frac{A_{max,tr}}{A_{max,ss}}} = \sqrt{2}\,\sqrt{\frac{1}{\sqrt{3}}} = 1.075$$

Notice that the condition $T_{ss} = T_{tr}$ is obtained with a torque $\tau_{ss} = 2\tau_{tr}$ ($A_{max,ss} = 2A_{max,tr}$).

# c - Frequency analysis

In many motion control applications, when inertial loads have to be considered, the frequency range interested by the acceleration (torque) profile of the trajectory should be limited in order to avoid resonances or unmodeled dynamics of the mechanical structure.

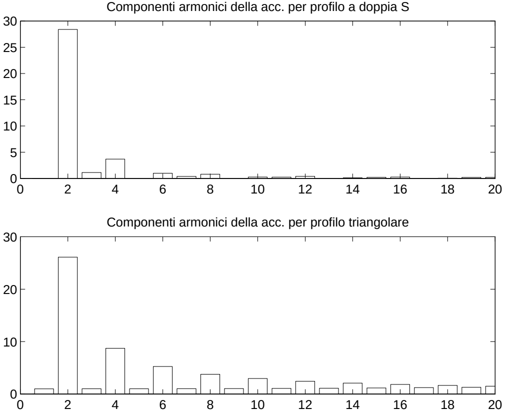
This aspect, that should always be taken into consideration, is of particular relevance in case of mechanisms with structural elasticities or high inertia. It is then important to evaluate the frequency content of the torque signals in order to understand their influence on the mechanical structure.

With this respect, it is obvious that the smoother the profiles, the better the results are (e.g. the double S trajectory is better that the triangular one because the frequency range is narrower).

# c - Frequency analysis

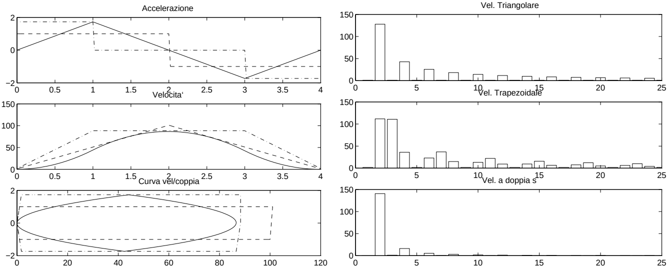
The figure, obtained with accelerations $A_{max,tr} : A_{max,ss} = 1 : \sqrt{3}$, it is possible to notice that the frequency content of the double S profile is lower (already in the second harmonic) than the triangular profile

The frequency range interested by the double S profile is narrower, and then its effect on resonances and unmodeled dynamics of the mechanical system (if present) is reduced.



# c - Frequency analysis

Another example, considering a trapezoidal profile.




In case even double S trajectories are not smooth enough and oscillations are generated on the mechanical structure, smoother profiles should be adopted like, for example, trajectories with a trapezoidal jerk profile.

More in general, motion profiles with derivative up to a given order $n$ should be considered: with a trapezoidal (triangular) velocity the trajectory is a $C^1$ function (continuous first derivative, while the second derivative is discontinuous), a double S trajectory is a $C^2$ function, etc.

An alternative approach is to use Spline functions with a proper order.

# Coordination of more motion axes

In many applications, many motion axes are present and need to be coordinated or synchronized. It is therefore necessary to take proper actions for this purpose, ranging from a simple synchronization of the start/stop instants to more complex operations.

Example: automatic machine for packaging medicines (pills)

- *450 motion axes:*
  - 150 electric drives (DC, brushless),
  - 300 step motors
- grouped in *40 blocks* to be synchronized and coordinated
- *specific packages for the single client* (how many pills, what time, ...)

# Coordination of more motion axes

## Example: automatic machine for lifting TGV trains

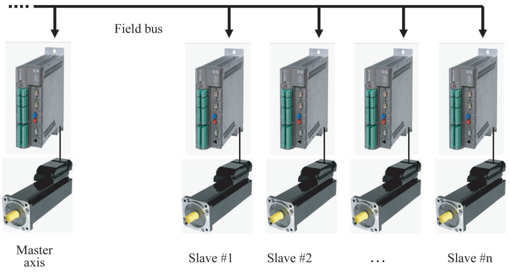

- Trains up to *200 meters* long
- Weight up to *386 tons*
- Accuracy between the two extremities: *1 mm*
- N. of lifting stations: 13

# Coordination of more motion axes

In multi-axis machines based on mechanical cams, the synchronization of the different axes of motion is simply achieved by connecting the slaves to a single master (the coordination is performed at the mechanical level).

In case of electronic cams the problem must be considered in the design of the motion profiles for the different actuators (the synchronization is performed at the software level).

A common solution is to obtain the synchronization of the motors by defining a master motion, that can be either virtual (generated by software) or real (the position of an actuator of the machine), and then by using this master position as "time" (i.e. the variable $\theta(t)$) for the other axes.

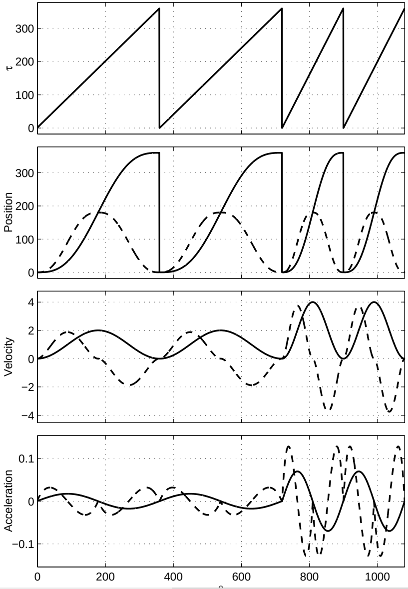


# Coordination of more motion axes

An example is reported in the figure, where the variable $\tau$ is computed as a function of the angular position $\theta$ of the master. In the first two cycles ($\theta \in [0^0, 720^0]$) the motion is "slow" ($\tau = \theta$), while in the last one ($\theta \in [720^0, 1080^0]$) the motion is "fast" ($\tau = 2\,\theta$).

Two slave axes are present: the first one generates a cycloidal profile from $q_{c0} = 0^0$ to $q_{c1} = 360^0$ (solid), while the second one generates a polynomial profile of degree 5 (dashed) interpolating the points $q_{p0} = 0^0, q_{p1} = 180^0, q_{p2} = 0^0$, in both cases for $\tau = [0^0, 360^0]$.

Note that in the last cycle the velocity values are doubled, while the accelerations are four times those present in the the first cycles.



# Coordination of more motion axes

In defining the (constant) velocity $v_c$ of the master axis, i.e. the motion law $\theta(t)$, the most 'stressed' axis (in terms of velocity, acceleration, ...) should be taken into consideration in order to define profiles that can be generated by each motor:

$$
\begin{aligned}
v_c \quad = \quad \min \Bigg\{ & \frac{v_{max\,1}}{|\dot{q}_1(\theta)|_{max}}, \dots, \frac{v_{max\,n}}{|\dot{q}_n(\theta)|_{max}}, \sqrt{\frac{a_{max\,1}}{|\ddot{q}_1(\theta)|_{max}}}, \dots, \sqrt{\frac{a_{max\,n}}{|\ddot{q}_n(\theta)|_{max}}}, \\
& \sqrt[3]{\frac{j_{max\,1}}{|\dddot{q}_1(\theta)|_{max}}}, \dots, \sqrt[3]{\frac{j_{max\,n}}{|\dddot{q}_n(\theta)|_{max}}} \Bigg\}
\end{aligned}
$$

Synchronization of different axis of motion can also be defined analytically, as already briefly discussed for trapezoidal velocity or spline trajectories.