On DC-link voltage stabilization of shunt active filters

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Abstract: In this paper the control of DC-link voltage in shunt active filters is studied. The main control objective in this kind of active power filters is to inject a proper current in the electric mains in order to compensate for harmonic distortion generated by nonlinear loads. According to this purpose, the proposed stabilizing algorithm allows the DC-link voltage to properly oscillate within the admissible range when load currents are compatible with the energy stored in the capacitor. Differently, when a transient disturb pushes the DC-link voltage near the bounds, the controller increases its action in order to avoid unnecessary turn-off by hardware/software protection. This solution leads to an improved reliability of the shunt active filter.

Key-words: Active power filters, AC/DC converters, DC-link voltage regulation

1 Introduction

The use of electric nonlinear devices, generates harmonics in voltage and current mains spectra. This phenomenon can determine additional power losses and the risk of equipment damage or malfunctioning. Hence some countermeasures have to be taken to reduce this harmonic distortion. Current harmonics have been traditionally compensated with passive filters, which have several drawbacks: their operation depends on the network impedance, they have to be tuned on fixed frequencies, etc. In the last decades, the fast development of power electronics components and control processors has led to the introduction of the so-called Active Power Filters (APF) [1],[6]. These devices can be more expensive than the passive filters, but they are less network-dependant and can be tuned on different frequencies changing a few software parameters.

A particular kind of APF are the Shunt Active Filters, whose purpose is to inject into the mains wires a proper current in order to compensate partially or totally for the harmonic current generated by nonlinear loads. Shunt active filters are based on AC/DC boost converter topology and their performances are determined by: (a) the converter parameters, (b) the method to compute current references and (c) the control algorithm adopted. The generation of the current reference is typically based on the detection of the harmonic content of load currents by means of different techniques as the instantaneous power theory [4], the time-domain correlation techniques [8], the FFT, etc. The control of the current produced by the shunt active filter is deeply analyzed in literature, while an aspect usually not well highlighted is the DC-link voltage control. Focusing the control objective only on current harmonics compensation, the DC-link voltage dynamics becomes a stable but oscillating internal dynamics for the system. A proper selection of the DC-link capacitor guarantees voltage oscillation inside an admissible interval for a given range of load harmonics [7]. However, load current transients and parasitic effects can make DC-link voltage to leave its allowed interval, requiring the software/hardware protections to stop the filter operations.

The objective of this paper is to provide a voltage stabilizing method which is compatible with the current compensation objective and can face conditions as load current transients avoiding halts of the power filter. In order not to impair the harmonics compensation, the voltage controller minimizes its action in
normal load current conditions; while, when overload occurs, the controller significantly changes the filter current in order to keep the DC-link voltage into its admissible range. This behavior has been achieved applying Input to State Stability (ISS) approach [3] with a suitable shaping of the voltage controller asymptotic gain.

The paper is organized as follows. In the first section the model of the shunt active filter is shown, in the second one the nominal behavior and the admissible range of the DC-link voltage are described. In section 4 the proposed control algorithm is presented. This section is focused on voltage control, while the current control is just sketched. Simulation results are reported in section [5].

2 Model

The scheme of the active filter considered in this article is presented in Fig.1. It is a three-phase AC/DC boost converter, where the capacitor is the main energy storage element and the inductors are used for the control of the filter currents by means of the converter voltages. In figure (1): \( v_{ma}, v_{mb}, v_{mc} \) are the mains voltages, \( i_{ma}, i_{mb}, i_{mc} \) are the mains currents, \( i_{la}, i_{lb}, i_{lc} \) are the load currents, \( i_a, i_b, i_c \) are the filter currents, \( v \) is the capacitor voltage, \( L \) is the value of the inductances, \( C \) is the value of the DC-bus capacitor.

The mains voltages are co-sinusoidal of frequency \( \omega_m = 2\pi f_m, f_m = (50 \text{ Hz or } 60 \text{ Hz}) \), balanced and equilibrated.

\[
\begin{align*}
v_{ma}(t) & = V_m \cos(\omega_m t) \\
v_{mb}(t) & = V_m \cos(\omega_m t - \frac{2\pi}{3}) \\
v_{mc}(t) & = -(v_{ma}(t) + v_{mb}(t))
\end{align*}
\]

The load currents \( i_a, i_b, i_c \) are balanced and periodic of frequency \( f_m \):

\[
\begin{align*}
i_{ij}(t) & = I_{j0} + \sum_{k=1}^{M} I_{jk} \cos(\omega_m t + \theta_{jk}) \\
j & = a, b \\
i_{lc}(t) & = -i_{la}(t) - i_{lb}(t)
\end{align*}
\]

The filter model can be defined, starting from inductor dynamics and neglecting parasitic resistances

\[
v_{m}(t) - L \frac{d\mathbf{v}(t)}{dt} = \mathbf{u}_{xyz}(t) v(t) - v_{NK}(t) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
\]

where \( \mathbf{v}_m(t) = [v_{ma}(t), v_{mb}(t), v_{mc}(t)]^T \), \( \mathbf{i}(t) = [i_a(t), i_b(t), i_c(t)]^T \), \( \mathbf{u}_{xyz}(t) = [u_x(t), u_y(t), u_z(t)]^T \) are arrays representing, respectively: mains voltages, filter currents, control inputs of the six-switches-bridge. In particular, a PWM control [2] is assumed for the switches, hence \( u_x, u_y, u_z \in [0, 1] \). From the sum of the three scalar equations above, it follows that

\[
v_{NK}(t) = \frac{u_x(t) + u_y(t) + u_z(t)}{3} v(t)
\]

Defining

\[
\mathbf{u}_{abc}(t) = \mathbf{u}_{xyz}(t) - \frac{u_x(t) + u_y(t) + u_z(t)}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
\]

and considering the dynamics of capacitor \( C \), the complete filter model in the a-b-c reference frame can be written as follows:

\[
\begin{align*}
\frac{d\mathbf{i}}{dt} & = -\frac{1}{L} \mathbf{v}_{abc}(t) + \frac{1}{L} \mathbf{v}_m(t) \\
\frac{dv}{dt} & = \frac{1}{C} \mathbf{u}_{abc}(t) \mathbf{i}(t)
\end{align*}
\]

where all the arrays are balanced (i.e. for each vector \( \mathbf{x} = [1, 1, 1] \) \( \mathbf{x} = 0 \)). This property allows to represent equations (1) and (2) in a classical two-phase Space
Vector [2] reference frame. In particular, it is useful to consider a d-q synchronous reference frame, aligned to the mains voltages. The matrix that allows to describe the three-phase balanced signals of (1), (2) in this bi-dimensional reference frame is:

\[
d d_q T_{abc}(t) = k_c \begin{bmatrix} C_{\alpha 0} & C_{\alpha t - 2\pi/3} & C_{\alpha t + 2\pi/3} \\ -S_{\alpha 0} & -S_{\alpha t - 2\pi/3} & -S_{\alpha t + 2\pi/3} \end{bmatrix}
\]

where \(C_x = \cos(x)\), \(S_x = \sin(x)\), \(k_c\) is a suitable positive constant.

In the new coordinates, the filter equations (1), (2) become

\[
\begin{align*}
\frac{d\mathbf{i}}{dt} &= \begin{bmatrix} 0 & \omega_m \\ -\omega_m & 0 \end{bmatrix} \mathbf{i}(t) - \frac{v(t)}{L} \mathbf{u}(t) + \frac{\mathbf{v}_m}{L} \\
\frac{dv}{dt} &= \frac{2}{3k_c^2 C} \mathbf{u}^T(t) \mathbf{i}(t)
\end{align*}
\]

where

\[
\begin{align*}
\mathbf{v}_m &= \frac{3 k_c}{2} V_m \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} V_{m0} \\ 0 \end{bmatrix} \\
\mathbf{i}(t) &= \begin{bmatrix} i_{ld}(t) \\ i_{ql}(t) \end{bmatrix} = \begin{bmatrix} i_{ld0} + i_{ldh}(t) \\ i_{ql}(t) \end{bmatrix} \\
\mathbf{u}(t) &= [u_d(t), u_q(t)]^T \\
\mathbf{i}(t) &= [i_d(t), i_q(t)]^T
\end{align*}
\]

**Remark:** according to the admissible range for \(u_d, u_q, u_c\), the vector \(\mathbf{u} = d_q T_{abc} u_{abc}\) must be included into the hexagon reported in Fig.2.

\[0.8 \times 0.8\]

**Figure 2:** Hexagon of feasible \(u_{abc}\)

3 Nominal behavior of the DC-link voltage

The main control objective of a shunt active filter is to cancel all or a part of the reactive and harmonic current of the load. Hence the reference \(i^*\) for the filter current \(i\) is a suitably defined function of the load current \(i_L\). For instance, if a full harmonic and reactive compensation is needed, then

\[
\mathbf{i}^* = \begin{bmatrix} i_{ld}^* \\ i_{q}^* \end{bmatrix} = - \begin{bmatrix} i_{ld0} - i_{ld} \\ i_{ql} \end{bmatrix}
\]

where \(i_{ld0}\) is the mean value of the active component \(i_{ld}\). In order to fully achieve this purpose, the DC-link voltage \(v(t)\) has to oscillate. The necessity of this behavior is clear considering the energy balance equation

\[
C \frac{dv^2}{dt^2} = \frac{2}{3k_c^2} \left[ \mathbf{v}_m \mathbf{i}(t) - L \frac{d}{dt} \frac{d^2 v}{dt^2} \right]
\]

and taking into account the periodic behavior imposed to the filter currents.

On the other hand, the instantaneous value of \(v(t)\) must be upper and lower bounded, as deeply discussed in [7]. The upper bound is related to the maximum admissible voltage for the capacitor \(C\), while the lower bound depends on current controllability constraints, as briefly reported in the following. Considering the (3) and substituting the current \(i\) with its reference \(i^*\), it follows that

\[
\| \mathbf{u}^*(t) \| = \frac{L}{v(t)} \| \begin{bmatrix} 0 & \omega_m \\ -\omega_m & 0 \end{bmatrix} \mathbf{i}^*(t) + \frac{\mathbf{v}_m}{L} - \frac{d\mathbf{i}^*}{dt} \|
\]

\[
\leq \frac{L}{V_{min}} \| \begin{bmatrix} 0 & \omega_m \\ -\omega_m & 0 \end{bmatrix} \mathbf{i}^*(t) + \frac{\mathbf{v}_m}{L} - \frac{d\mathbf{i}^*}{dt} \|
\]

In order to ensure that this control vector is included into the hexagon of Fig.(2), the voltage bound \(V_{min}\) must be sufficiently high. Moreover, from a practical point of view, \(V_{min}\) must be also greater than a minimum value to guarantee the correct behavior of all the electronic devices, as the system control board whose supply is usually derived from the DC-link capacitor.

As reported in [7], the belonging of voltage \(v\) to the allowed region is mainly related to the correct selection of capacitor value \(C\) with respect to the nominal harmonic load to be compensated. Hence, the DC-link voltage control action must be focused on the satisfaction of voltage constraints when transitory overload conditions occur.
4 Control algorithm

Consider the model (3),(4). In order to make linear and independent the current dynamics, the following control $u$ is chosen

$$ u = \frac{1}{v(t)} \left\{ -\omega_m L i(t) + v_m - \xi(t) \right\} $$

where $\xi$ is an auxiliary control variable. Defining the current error vector

$$ \tilde{i} = i(t) - i^*(t) $$

and considering (5), the current error dynamics is

$$ \frac{d\tilde{i}}{dt} = \frac{\xi(t)}{L} + d^*(t) $$

where

$$ d^* = -\frac{di^*}{dt} $$

The auxiliary control $\xi(t)$ must be a function of $\tilde{i}$ suitably chosen in order to stabilize $\tilde{i}$ and to reject the disturbance $d^*(t)$. Since this paper mainly concerns the DC-link voltage stabilization, the detailed analysis of the current controller is omitted and it is assumed that the auxiliary control $\xi(t)$ is “fast enough” to produce a negligible $\tilde{i}$ with respect to the voltage dynamics (this assumption is usually admissible owing to the large value of $C$). Hence considering (4),(5), the voltage dynamics is the following:

$$ \frac{dv^2}{dt} = \frac{2}{3k_w^2C} \left[ v_d(t) i_d^* + v_q(t) i_q^* \right] $$

where $[v_d(t), v_q(t)]^T = v(t)$ are the voltage imposed to control $i_d(t)$, $i_q(t)$.

The control objective for the DC-link voltage is twofold:

- To guarantee the voltage boundness $V_{\text{min}} \leq v(t) \leq V_{\text{max}}$ for a given maximum current.
- To perform a low control action when the voltage $v(t)$ is inside the admissible range $[V_{\text{min}}, V_{\text{max}}]$. In this way, the main current control objective is not impaired.

The strategy adopted to obtain voltage control objective is based on the introduction of an additional part $i_{dv}^*$ in the reference currents:

$$ i^* = \begin{bmatrix} i_{d}^* \
 i_{q}^* \end{bmatrix} = - \begin{bmatrix} i_{dL}^* \
 i_{qL}^* \end{bmatrix} + \begin{bmatrix} i_{dv}^* \end{bmatrix} $$

where $i_{dL}^*$, $i_{qL}^*$ are the original reference components derived from the load currents. In order to design the voltage control, let define the variable

$$ \bar{e} = \frac{v^2(t)}{2} - e^* $$

with $e^* = 0.5(V_{\text{max}}^2/2 + V_{\text{min}}^2/2)$ constant value around which $e(t)$ has to oscillate. The resulting $\bar{e}$ dynamics is the following:

$$ \dot{\bar{e}} = [v_d(t) i_{dvL} + d_e(t)] $$

where $d_e$ has been defined as

$$ d_e(t) = v_d(t) i_{dvL}^* + v_q(t) i_{qL}^* $$

The first step to fulfill the control objectives is to guarantee the Input-to-State Stability of (9) with respect to the disturbance $d_e(t)$. Choosing the control action as

$$ i_{dvL}^* = -\text{sgn}(\bar{e})f(|\bar{e}|) $$

and defining, according to (5):

$$ v_d = V_{md0} + v_{dl}(t), $$

the following dynamics is derived:

$$ \dot{v}_d = \frac{2}{3k_w^2C} \left( [V_{md0} + v_{dl}(t)] \text{sgn}(\bar{e})f(|\bar{e}|) - d_e(t) \right) $$

Let consider the following Lyapunov candidate function

$$ W(\bar{e}) = \frac{1}{2} \bar{e}^2 $$

It follows that

$$ \dot{W} = \frac{2}{3k_w^2C} \left( [V_{md0} + v_{dl}(t)] f(|\bar{e}|) \bar{e} - d_e(t) \bar{e} \right) $$

Assuming that the current control voltage $v_{dl}$ is bounded as follows

$$ |v_{dl}(t)| \leq V_d < V_{md0} $$
it results
\[ \dot{W} \leq -\frac{2}{3k^2C} \left\{ [V_{md0} - V_d]f(|\varepsilon|) |\varepsilon| - |d_v(t)||\varepsilon| \right\} \]

Defining \( k_0 > 0 \) and choosing
\[ f(|\varepsilon|) = \frac{1}{V_{md0} - V_d} \left[ \frac{3k^2C}{2} k_0 |\varepsilon| + f_1(|\varepsilon|) \right] \]  \hspace{1cm} (11)

where \( f_1 \) is a class-K function of \( |\varepsilon| \) it follows that
\[ -\frac{2}{3k^2C} \left\{ [V_{md0} - V_d]f(|\varepsilon|) |\varepsilon| - |d_v(t)||\varepsilon| \right\} < -k_0 |\varepsilon| \]
\[ \forall |\varepsilon| > f_1^{-1}(|d_v(t)|) \]. Hence the controlled voltage dynamics is ISS with respect to \( d_v(t) \).

The second step to achieve the control objectives is devoted to a suitable shaping of \( f_1(|\varepsilon|) \). The result is shown in Fig.3. For error amplitude below \( e_{th} \) a low control action is performed: the voltage oscillates as required for correct current compensation. When error modulus is greater than \( e_{th} \) the control effort is increased to “save” the DC-link voltage, even if this action will impair the filter performances in terms of harmonic compensation. Note that the maximum slope of \( f_1(\cdot) \) is limited by the dynamic separation imposed between current and voltage control and by the bandwidth limits determined by actual discrete-time implementation. The effect of \( f_1(\cdot) \) in terms of “allowed oscillation” is better depicted in Fig.4 where the asymptotic gain \( \gamma(\cdot) = f_1^{-1}(\cdot) \) of the voltage dynamics is reported.

In Fig.3, \( e_{max} \) indicates the maximum acceptable error amplitude. When \( |\varepsilon| \) exceeds that limit, the software/hardware protections act to halt the filter operation.

Remark: In this analysis the effects of inductance parasitic resistances and electronic devices supply current are neglected. These are dissipative components which produce a slow decrease of the DC-link voltage. In actual implementation an integral action can be added to the DC-link controller to compensate for this effect.

5 Simulation results

The shunt active filter parameters adopted in simulations are: \( L = 3 \) mH, \( C = 200 \) \( \mu \)F. The distorted load current which must be compensated is shown in Fig.5, in the d-q reference frame. In the d-component a step transient is considered: a constant value of 3 A is added at time \( t = 0.1 \) s and it is removed at time \( t = 0.2 \) s.

In order to enlighten its stabilization properties, the proposed solution has been compared with a linear proportional controller:

\[ i_{dv}^* = -k_{min}\varepsilon \]

where \( k_{min} \) is equal to the minimum slope of the \( f(|\varepsilon|) \) described in (11). In Fig.6 the performances of the two solutions are reported. Both controllers produce a small control action \( i_{dv}^* \) in steady-state condition, this gives a negligible perturbation.
of compensation properties of the filter. Differently, during the load transient, the proportional controller is not able to guarantee a “safe” DC-link voltage (i.e. $v^2(t)/2$ inside the interval $[2.2 \cdot 10^3, 4.1 \cdot 10^3]$), while the proposed one achieves the desired behavior, increasing the control action amplitude.

Differently, it is able to avoid unnecessary halts by hardware/software protections if a transient disturb occurs.

### References


### 6 Conclusions

A shunt active filter DC-link voltage regulator has been presented. It does not significantly affects the compensation performances when load currents are compatible with the energy stored into the capacitor.