Tendon-sheath transmission systems are used in various robotic applications, such as robotic hands [1], [2], because very often it is not possible to place the actuators inside the fingers. The use of pulleys increases the mechanical complexity and the dimension of the structure and implies that the tendons must be properly preloaded, while the use of sheaths reduces the size, the complexity, the cost and at the same time increases the reliability of the overall system.

This transmission modality introduces some side effects due to the tendon compliance and to the friction between tendon and sheath. These phenomena introduce deadzone, hysteresis and direction-dependent behavior in the force control [6].

A simple tendon-sheath static model that highlight nonlinearities and direction-dependent behavior is presented in [6], [7]. Starting from this model, we have made some additional considerations about environment disturbance and tension distribution along the tendon. The environment is modeled as a mass linked to a mobile constrain by a spring. The disturbance is represented by the movement of the constrain and by an external force applied to the environment mass. This model is adequately modified to consider the effect of non-constant preload and environment disturbance.

In [7], a lumped parameters dynamical model is used to validate the input-output relation derived from the experiments. We use a similar model in simulation to study the tension distribution inside the tendon and the effect of the environment load. In our work, we show that this model has a strange behavior in stationary condition due to the considered static friction model (Coulomb friction). The use of a dynamical friction model (Dahl model [8]) corrects this drawback.

In order to compensate the nonlinear behavior of the driving system, a suitable controller with a friction compensator is designed. The friction compensation law presents discontinuities when the sign of the tendon velocity changes, so the bandwidth of the actuators must be very high to obtain good performance. This switching control action introduces dynamics behaviors in the system, such as vibrations in the tendon, that are not considered in the static tendon model. Simulative results show that this controller is able to compensate both friction effects and also the environment disturbances.

II. Transmission Characteristics

The static model of the tendon-sheath driving system presented in [6], [7] is (see Fig. 2):
is the Coulomb friction coefficient. Assuming that \( \dot{\epsilon} \) has the same sign along the whole length of the tendon, and neglecting the effect of stiction, we can model an infinitesimal tendon element as:

\[
N = T \frac{d\gamma}{dx} = T \frac{dx}{R} \quad (1)
\]

\[
dT = -F_f = -\mu T \frac{dx}{R} \text{sign}(\dot{\epsilon}) \quad (2)
\]

where \( d\gamma \) is the angle subtended by the arc of length \( dx \) and \( R \) is the radius of the sheath curvature. From these equations we can write:

\[
\frac{dT}{dx} = \left[ -\frac{\mu}{R} \text{sign}(\dot{\epsilon}) \quad 0 \right] \begin{bmatrix} T \\ \delta \end{bmatrix} + \left[ 0 \quad -\frac{1}{EA} \right] T_0
\]

or, in more compact form

\[
\frac{d\theta}{dx} = D\theta + \Delta T_0 \quad (3)
\]

where \( T_0 \) is the tendon preload. The solution of this system is:

\[
T(x) = \begin{cases} 
T_{in} \exp\left[-\frac{\mu}{R} \text{sign}(\dot{\epsilon}) \right] & \text{if } x < L_1 \\
T_0 & \text{if } x \geq L_1
\end{cases} \quad (4)
\]

\[
\delta(x) = \begin{cases} 
H(x) - T_{in}x + \frac{\mu}{R} T_{in} \text{sign}(\dot{\epsilon}) & \text{if } x < L_1 \\
H(L_1) - T_0L_1 + \frac{\mu}{R} T_{in} \text{sign}(\dot{\epsilon}) & \text{if } x \geq L_1
\end{cases} \quad (5)
\]

where

\[
H(x) = -\frac{R}{\mu} T_{in} \text{sign}(\dot{\epsilon}) \exp\left[-\frac{\mu}{R} x \text{sign}(\dot{\epsilon}) \right] \quad (6)
\]

and

\[
L_1 = \min\{x \in T(x) = T_0\}
\]

When \( L_1 \) becomes equal to \( L \), variation of the input tension is immediately transmitted to the output side, with a ratio of:

\[
\frac{dT_{out}}{dT_{in}} = e^v \text{ if } \dot{\epsilon} > 0
\]

This implies that \( \dot{\epsilon} \) has the same sign along the tendon, positive during the pulling phase and negative during the loosening one.

This model is obtained by considering only the static Coulomb friction. With a dynamical friction model, in our case the Dahl model (see Fig. 3), it is possible to see that the slope of the exponential term of the (4) can assume all the intermediate values between \( v \) and \(-v\). To explain this phenomenon, we can note that in (2), the variation of the tendon tension is due only to the Coulomb friction. In the Dahl model, the value of friction can vary between \( F_c \) and \(-F_c\), where \( F_c \) is the absolute value of the Coulomb friction.

\[
\frac{dT_{out}}{dT_{in}} = e^v \text{ if } \dot{\epsilon} < 0
\]

where

\[
v = \frac{\mu}{R} L
\]

III. TENDON DYNAMICAL MODEL

A lumped parameters tendon model similar to the one presented in [7] is used for simulation. The equilibrium of each tendon element is given by:

\[
m_i \ddot{\epsilon}_i - c_i (\dot{\epsilon}_{i+1} - 2\dot{\epsilon}_i + \dot{\epsilon}_{i-1}) = T_{i+} + T_{i-} - F_{f_i}
\]

\[
T_{i+} + T_{i-} = k_i (\epsilon_{i+1} - 2\epsilon_i + \epsilon_{i-1})
\]

\[
F_{f_i} = \frac{\mu}{2R} (T_{i+} - T_{i-}) \text{tanh}(k_i \epsilon_i) = \frac{\mu}{2R} k_i (\epsilon_{i+1} - \epsilon_{i-1}) \text{tanh}(k_i \epsilon_i)
\]

where \( m_i, \epsilon_i, c_i, F_{f_i}, T_{i+}, T_{i-}, k_i \) and \( k_i \) are respectively the mass, the position, the damping, the friction, the input side.
tension, the output side tension, the friction parameter in the Karnopp model and the stiffness of each tendon element (see Fig. 4).

Note that the $\text{sign}(\cdot)$ function in the Coulomb friction model is replaced in the Karnopp model by the $\tanh(\cdot)$ function, eq. (9), to avoid numerical problems during simulation [8].

The simulation results highlight that in stationary conditions the tendon tension distribution shows a behavior without physical meaning, see Fig. 5. In particular, it is possible to note that the output tension increases without an input tension variation, as if the effect of friction is vanishing. This fact is due to the friction model used, that neglects the stiction effect. The use of the Dahl friction model [8] permits to take into account the effects of the friction also when the velocity goes to zero and for little movements. Now, the equilibrium of each tendon element is given by:

\begin{align}
\dot{v}_i &= \dot{v}_i \\
\dot{\tau}_i &= \frac{1}{m_i} \left[ c_i(v_{i+1} - 2v_i + v_{i-1}) + k_i (\epsilon_{i+1} - 2\epsilon_i + \epsilon_{i-1}) - F_{fi} \right]
\end{align}

\begin{align}
\dot{F}_{ai} &= k_b \left[ v_i - \frac{2nR F_{fi}}{\mu L} \frac{|v_i|}{k_i (\epsilon_{i+1} - \epsilon_{i-1})} \right]
\end{align}

where $k_b$ is the bristle stiffness characterizing the contact between tendon and sheath.

The choice of the this friction model is justified also by the simulation results, as it is possible to see in Fig. 6. Other dynamic friction models could be evaluated (e.g. LuGre model, Bliman and Sorine model etc. [8]) but an adequate parameters identification procedure is necessary to apply more sophisticated models.

The transmission characteristic obtained from simulations is reported in Fig. 7. In this plot, we can recognize four phases: pulling phase (A), relaxation dead zone (B), loosening phase (C) and stretching dead zone (D).

Figure 8 shows the distribution of tension along the tendon when a sinusoidal input is applied.

### IV. EXPERIMENTAL RESULTS

In order to validate the simulation results and also to characterize the friction parameters of different materials, for both tendon and sheath (alloy-alloy, Kevlar-alloy, Teflon-alloy, Teflon-Teflon, spectra-alloy, spectra-Teflon), some experiments have been conducted. In the laboratory setup, the tendon is connected at each end to an actuator, composed by a DC motor with encoder, gearheads and ballskrew. The input and output tensions are measured by two load cells.
Fig. 8. Simulation results: tendon tension distribution in the dynamic model with sinusoidal input.

![Tension distribution along the tendon](image)

placed at each tendon end. The sheath curvature radius $R$ can be changed, replacing the black plastic cylinder visible in Fig. 10, and the possible bending angles $\gamma$ are $\pi/2$, $\pi$ and $3\pi/2$.

The experimental results in Fig. 11(a) and Fig. 11(b) confirm that the transmission characteristic (alloy-alloy) does not depend on the tendon length, but only on the bending angle $\gamma$. The parameters of friction can be determined from the transmission characteristic (see Fig. 12(a)). The comparison between simulation and experimental results (see Fig. 12(b)) validates the proposed model.

From Fig. 12(b) it is also possible to note that for high tendon tension, the compliance of the sheath is not negligible. A model that takes into account also the elasticity and the dynamics of the sheath is under development.

V. TENDON TRANSMISSION CONTROL

The main target of this research is to control the output tension of the tendon compensating friction and elasticity effects, and avoiding some typical side effects like limit cycles [9], [10], that classic PID controller introduces in this kind of nonlinear systems. For this purpose, a friction compensator is designed so that a suitable action is added to the input tension in order to obtain the desired value for the output tension, eq. (4).

The friction compensation law is not based on the tendon velocity, but on the tension tracking error because if the environment constraint is considered fix, a positive tracking error means that the tendon must be pulled and then the tendon velocity must become positive and vice versa. So the control action $F_c$ is:

$$F_c = T_s \exp \left[ \frac{\mu d L}{R} \text{sign}(e) \right]$$  \hspace{1cm} (13)

where $T_s$ is the tension setpoint and $e$ is the tension tracking error.

In Fig.13(a) the response of the tension control system is
reported. In Fig.13(b) it is possible to see that the tracking error is very small also with sinusoidal setpoint, but presents fast variations due to the switching behavior of the friction compensation action (see also Fig.14(a)). In Fig. 14(b) the tendon input and output tensions are reported.

The friction compensation law described by eq.(13) is very similar to a sliding mode controller, because the control action switches from a maximum to a minimum value related to the magnitude of the disturbance (friction). Note that in eq.(13) if the parameters $R$ and $L$ are constant the exponential term may be computed only once both for positive and negative tracking error.

In order to reduce the error chattering, a proper boundary layer is introduced. The new friction compensation action is:

$$F_c = \begin{cases} T_s \exp \left[ \frac{\mu_d}{R} L \text{sign}(e) \right] & \text{if } |e| \geq e_{th} \\ T_s \left( 1 + \frac{|e|}{e_{th}} \exp \left[ \frac{\mu_d}{R} L \text{sign}(e) \right] \right) & \text{if } |e| < e_{th} \end{cases}$$

where $e_{th}$ is the amplitude of the boundary layer.

The response of this controller is reported in Fig. 15(a) and Fig. 15(b). From these Figures it is possible to note that the boundary layer makes the tracking error much more smooth for fixed setpoint, see Fig.15(b) from $t = 0$ to $t = 2$. For $t > 2$ a variation on the environment disturbance is introduced, and this causes a significative growth of the tracking error.

By a suitable overestimation of the friction parameter, it is also possible to compensate for the effects of an environment disturbance. This is done by increasing the magnitude of the control action computed in eq.(14) with a suitable choice of the friction coefficient $\mu_{id}$. Fig.16(a) and Fig.16(b) report the response and the tracking error with a sinusoidal setpoint starting at time $t = 4.8s$ and a sinusoidal environment constrain motion.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, static and dynamic model of the tendon-sheath driving system are presented. A tendon tension sliding mode controller for the friction compensation is applied to this transmission system and some simulation results are reported. These results show that the output tension of the tendon can be controlled with a limited tracking error even in presence of environment disturbance and without introducing any limit cycle.
Fig. 16. Setpoint, output tension (a) and tracking error (b) with the overestimation of the disturbance.

The experiments on the tendon-sheath driving system is not concluded yet: in particular the control algorithm has not been tested on the experimental setup. In order to improve the reliability of the simulations and to take into account various friction phenomena, a more complex dynamic friction model may be considered, i.e. the LuGre model.

In a tendon-sheath driving system, a fundamental rule is played by the sheath, whose dynamics is neglected at the moment. A simple model of the sheath that takes into account its finite stiffness is under development.

The dynamics of the nonconstrained parts of the sheath is very important because the mass of the sheath is many times larger than the mass of the tendon, and this may affect the force transmission properties of the system. A tendon-sheath PDE model is under development. Starting from this model it will be possible to use the infinite dimension port-Hamiltonian approach to design the controller.

In the next phase of this analysis, also the dynamic model of the finger will be included to evaluate the performance of the overall finger control. To implement the impedance control of the robotic finger, an output tendon stiffness controller will be designed.

REFERENCES