Abstract—Automated manual transmissions, equipped with servo-actuated clutch and gearbox, are becoming spread solutions in the European car market, thanks to the achievable improvement of comfort and performances. Driveline dynamics shows an intrinsically hybrid behavior, due to interconnection of continuous-time mass-spring-damper dynamics of transmission shafts and “discrete” evolution imposed by clutch and gearbox actuators. In the formal framework of hybrid system modelling, a driveline model taking into account the main phenomena occurring during gear shifting is proposed in this paper. Main purpose of this work is to propose a driveline model, based on fundamental axle dynamics and abstraction of clutch and gearbox actuators; the proposed model is simple but at the same time it is sufficiently accurate to be used for the design of controllers for gear shift operations. Model validation has been performed on experimental data.

I. INTRODUCTION

Automatic gear shifting improves vehicle performance and efficiency and increases passengers and driver comfort. For these reasons, servo-actuated clutch and gearbox are becoming spread solutions in the European commercial car market. Objective of the supervising controller for automatic gear shift is to achieve fast and precise gear transitions, minimizing the possible oscillating responses of the driveline and maximizing the mean acceleration of the vehicle during the maneuver. Various solutions for driveline control have been addressed [1], [2]. Due to high order, complexity and nonlinearity of the driveline combined with the severe control requirements, a model-based approach for the controller design is required. Precise and detailed physically-based modelling of the driveline has been developed for system analysis and as a benchmark for controller performance testing [3]–[6]. On the other hand, a model which captures the main phenomena occurring during the gear shift and is sufficiently simple for the derivation of the control strategy is demanded for control design purposes.

The driveline of an automated manual transmission consists of the engine, transmission shafts, wheels and actuators for clutch and gearbox control. Both engine torque, clutch disk position and gearbox actuator commands (for speed and neutral phase selection, etc.) must be imposed in a coordinated way for gear shift control. The system is characterized by an intrinsically hybrid behavior: the mechanical system shows continuous mass-spring-damper dynamics, the clutch can be modelled with stick-slip phenomenon (Coulomb friction), while the gearbox introduces discrete dynamics corresponding to different speeds, neutral and gear synchronization phases, e.g. affecting equivalent inertias and speed ratios. Hence, a hybrid system approach, dealing with both continuous and discrete dynamics, is useful for modelling and control design purposes.

Extensive studies on hybrid system modelling approaches have been fulfilled [7]–[11]. In order to obtain a formal description of the driveline in the framework of hybrid systems, a new general hybrid model is presented in this paper. The proposed hybrid system takes into account both continuous-time and discrete event input/output signals with real and discrete values. Roughly speaking, the hybrid behavior consists in the fact that the continuous state evolves according to different dynamics dependent on the discrete states, while discrete transitions occur in accordance with the enabling of guard conditions dependent on continuous state evolution and/or input signals.

The aim of this work is to propose a hybrid model of a vehicle driveline for control design purposes. In particular, it focuses on mass-spring-damper dynamics of the shafts, combined with abstractions of the clutch and gearbox actuators, modelled by means of automata. All the control inputs involved in the gear shift (engine torque and clutch and gearbox commands) have been taken into account. For the driveline modelling, the proposed hybrid system approach has been exploited to describe the main phenomena occurring during gear shifting. It is useful both for the design of hybrid controllers supervising the automatic gear shift and for the evaluation of the performances achievable in the ideal conditions determined by the modelling assumptions and simplifications.

The structure of the paper is the following. In Section II a general framework for the modelling of hybrid systems is discussed. In Section III the proposed framework is utilized to model the vehicle driveline. In Section IV results on model validation performed on experimental data obtained during an automatic gear shift are discussed. Conclusions end the paper.

II. HYBRID SYSTEM MODEL

Before introducing the hybrid system model, some standard notations are recalled [10]–[12].

A. Notation

Given a set of valuations $V$ of a variable $v$, $v$ is called “continuous” if $V$ is a subset of the Euclidean space,
connected subset of the ordered metric time.

A continuous-time signal $s$ is such that $\text{dom}(s)$ is a connected subset of the ordered metric time $T$ (as a particular case, consider $\text{dom}(s) = T = \mathbb{R}^+$. A discrete-time signal $s$ is such that $\text{dom}(s) = \{t_i\}$ is a countable set, where $t_i \in \mathbb{R}^+$ represents the time of occurrence of the $i$-th event of the signal $s$. In order to consider signals which are not defined at certain time $t$, notation $v(t) = \emptyset$ denotes that $v$ does not assume any value at time $t$.

For any set $S$, $\mathcal{P}(S)$ denotes the power set of $S$, i.e. the set of the subsets of $S$. It is assumed that functions of time are piecewise-continuous from the left; given a signal $s$, the expression $s(t')$ denotes the successor of $s$, i.e. its right-hand limit.

Remark 2.1: It is worth noting that hybrid systems involve both continuous- and discrete-time signals, with real- and discrete-valued set of valuations. By means of properties of time axis and valuation sets, and exploiting the concept of event, it is possible to treat in an unitary way these types of dynamics. For example, continuous-time real-valued signals are state variables of continuous dynamics, while discrete-time signals can be associated to transitions between locations in finite state automata.

B. Hybrid system model

Hybrid system dynamics is roughly obtained by the combination of both continuous-time dynamics, described by ODEs, and by discrete event dynamical systems, described by automata. Continuous evolution can be considered as defined over disjoint time intervals, separated by event times associated to a transition between two automaton locations. Transitions can be enabled by evolution of continuous states in a certain domain and/or enforced by discrete-time input/internal signals. During transitions, reset of continuous states are performed and internal and output discrete events are generated.

Main feature of the proposed controlled hybrid system model is that input/state/output relations are enlightened, taking into account continuous-time and discrete event control inputs and outputs with real and discrete values. For concurrency of automata and for system composability, internal discrete events are also defined. Edges and guards (combined with invariant sets) are used for simple and clear modelling approach, which is useful when dealing with physical systems.

Definition 1: A Hybrid System is the dynamical system defined by the collection

$$H = (Q, X, U, Y, \Sigma_l, \Sigma_o, f, h, E, G, S_e, S_o, R, I, I_0)$$

where

- $Q \subseteq \{1, \ldots, N\}$ is the finite set of the discrete states (called also locations). $N$ is the number of the discrete states;
- $X = \mathbb{R}^n$ is the continuous state space;
- $U \subseteq \mathbb{R}^m \times U_D$ is the set of continuous and discrete-valued control inputs. $U_D = \prod_k U_{D,k}$, where $U_{D,k}$ are discrete sets;
- $Y \subseteq \mathbb{R}^p \times Y_D$ is the set of continuous and discrete-valued outputs. $Y_D = \prod_k Y_{D,k}$, where $Y_{D,k}$ are discrete sets;
- $\Sigma_l = \prod_k \Sigma_{l,k}$, where $\Sigma_{l,k}$ are finite sets, is the set of discrete-event control input;
- $\Sigma_e = \prod_k \Sigma_{e,k}$, where $\Sigma_{e,k}$ are finite sets, is the set of internal discrete-events;
- $\Sigma_o = \prod_k \Sigma_{o,k}$, where $\Sigma_{o,k}$ are finite sets, is the set of discrete-event output;
- $f : Q \times X \times U \rightarrow \mathbb{R}^n$ is the vector field of the continuous dynamics;
- $h : Q \times X \times U \rightarrow Y$ is the output map;
- $E \subseteq Q \times Q$ is the set of edges;
- $G : E \rightarrow \mathcal{P}(X \times U \times \Sigma_i \times \Sigma_o)$ is the guard condition;
- $S_e : E \times \Sigma_i \rightarrow \Sigma_o$ is the internal discrete-event map;
- $S_o : E \times \Sigma_i \rightarrow \Sigma_o$ is the output discrete-event map;
- $R : E \times X \times U \times \Sigma_i \times \Sigma_o \rightarrow X$ is the reset map.

Hybrid systems can be represented by means of arc-oriented graphs, where vertices are the discrete states $q \in Q$ and the edge $e = (q_i, q_f) \in E$ denotes an arc from the initial location $q_i$ to the final location $q_f$. The guard condition map $G(e)$ enables the corresponding edge $e$. As particular cases, guard conditions allow to deal with both autonomous and controllable transitions.

Autonomous transitions are defined by continuous-state dependent guards, independent of discrete-time signals. They are enabled by the evolution of the continuous state variables into regions of the continuous state-space or onto the invariant set boundary. Invariant sets define the domain of existence of continuous state for each location; hence, coherent definition of guard conditions must be assumed to force autonomous transitions.

Controlled transitions are defined by guards forced by input or internal discrete events.

The hybrid system dynamics is defined as follows.

Hybrid System execution. Time axis $T$ is partitioned into intervals $[t_i, t_{i+1}]$, $i \geq 0$, where $t_i \in T_f$ with $t_0 = 0$ (assuming time-invariant systems), and $t_{i+1} \geq t_i$. Set $T_f$ represents time instants when transitions are executed. Consider time variables $t \in T$, $t_i \in T_f$. Starting from an initial condition $(q_0, x_0) \in I_0$, given the continuous-time inputs $u : T \rightarrow U$ and the discrete-time inputs $\sigma_i : T_f \rightarrow \Sigma_l$, the hybrid system dynamics evolves as follows. Location remains constant at $q(t) = q$, while continuous
variables evolve according to the dynamics associated to location \( q \), namely \( \dot{x}(t) = f(q, x(t), u(t)) \), inside the invariant set of \( q \), i.e. with \( x(t) \in I(q, u(t)) \). Output variable is defined as \( y(t) = h(q, x(t), u(t)) \). Evolution within the location \( q \) ends when a guard of an edge starting from \( q \) is satisfied. An edge is enabled at time \( t_1 \) if there exists \( q' \in Q \) such that \( e = (q, q') \in E \) and it holds \((x(t_1), u(t_1), \sigma_e(t_1), \sigma_\nu(t_1)) \in G(e)\). At \( t_1 \), the transition \( q(t_1^+) = q' \) is instantaneously executed, resets are performed, i.e. \( x(t_1^+) = R(e, x(t_1), u(t_1), \sigma_e(t_1), \sigma_\nu(t_1)) \), and discrete-time internal and output signals are defined as \( \sigma_e = S_e(e, \sigma_e(t_1)) \), \( \sigma_\nu = S_\nu(e, \sigma_\nu(t_1)) \). From \( t \geq t_1 \), dynamics evolves in a similar way, with discrete state \( q' \).

### III. VEHICLE DRIVELINE MODEL

Vehicle driveline is composed by the elements which allow torque transmission from the engine to the wheels, namely engine, clutch, main shaft (connecting clutch and gearbox), gearbox, secondary shaft (connecting gearbox and differential gear), differential gear, axle shafts and wheels [3], [4], [6]. During gear shift, driveline behavior is imposed by acting on the engine torque and on the clutch and gearbox actuators.

For the considered modelling purposes, the engine can be thought as an ideal torque actuator with constant inertia, i.e. torque generation described by a detailed internal combustion engine model is not taken into account.

The purpose of the clutch is to transmit torque from the engine to the gearbox during normal operations. On the other hand, during gear shifting, when main shaft speed is required to change according to the requested gear ratio, clutch is used to decouple engine from main shaft. Torque transmissible by the clutch can be thought as the Coulomb friction torque through two rotating disks which can slip. Transmissible torque amplitude is a function of the normal force applied on the disks by a spring and an electro-hydraulic clutch actuator [5]. When the clutch is closed, a pre-load force keeps the disks in contact and the transmissible torque is much higher than the engine torque, thus avoiding clutch slipping. By modulating the actuator position, the transmissible torque is modified according to the so called clutch transmissibility curve. When the clutch is completely open, no torque can be transmitted and the engine is disconnected by the gearbox.

The gearbox considered consists of six couples of cogwheels permanently engaged each other; speed selection is performed by an electro-hydraulic actuator [6] which locks one couple of cogwheels to the shafts. When engaged gear, the speed of the main and secondary shafts are related by the corresponding gear ratio. When the gear is requested to disengage, the gearbox actuator applies a force in order to unlock the engaged cogwheels. When the torque transmitted by the gearbox is lower than a certain threshold, the gear is disengaged and the gearbox enters neutral state, i.e. main and secondary shafts are not connected. When the gear engagement is required, a friction torque is applied to the desired cogwheels by the synchronizer, thus dissipating kinetic energy. When the main and secondary shaft speeds present the correct ratio, cogwheels engage and shafts are connected.

According to previous considerations, in the framework of the hybrid system model presented in Section II, a driveline “control” model is proposed. As shown in Fig. 1, the continuous time dynamics is described by four inertial elements and a spring-damper dynamics for shaft flexibility, as:

\[
\begin{align*}
J_\omega \dot{\omega}_e &= T_e - T_e(q, x, u) \\
J_\omega \dot{\omega}_1 &= -b_1 \omega_1 + T_e(q, x, u) - T_{\text{gm}}(q, x, u) \\
J_\omega \dot{\omega}_2 &= T_{\text{gs}}(q, x, u) - T_{2w} \\
J_w \dot{\omega}_w &= T_{2w} - T_w \\
\dot{\theta}_{2w} &= \omega_w - \omega_w
\end{align*}
\]

where \( T_{2w} = k_{2w} \theta_{2w} + b_{2w}(\omega_2 - \omega_w) \) is the spring-damper torque of the secondary shaft. In (1), \( x = (\omega_e, \omega_1, \omega_2, \omega_w, \theta_{2w})^T \in \mathbb{R}^5 \) is the continuous state, constituted by angular speeds of engine \( \omega_e \), main shaft \( \omega_1 \), secondary shaft \( \omega_2 \), wheels \( \omega_w \) and secondary shaft angular displacement \( \theta_{2w} \). Discrete states for clutch and gearbox automata are defined as \( q = (q_e, q_1, q_2) \in Q = Q_e \times Q_1 \times Q_2 \), where input vector is \( u = (T_e, T_{\text{gm}}, T_{\text{gs}}, \theta_{\text{dsg}}, \theta_{\text{sync}}, T_{\text{w}})^T \in \mathbb{U} \). State and input signals will be defined in detail later. Continuous inputs \( T_e, T_{\text{gm}}, T_{\text{gs}} \) are torques transmitted by the clutch and by input and output gearbox shafts respectively, while the continuous-time discrete-valued variable \( r \) is the gear ratio; their dependence on the discrete state \( q \) of clutch and gearbox will be addressed later. \( J_e \) is the engine inertia, including crankshaft, flywheel and primary clutch disk; \( J_1 \) is the main shaft inertia, including secondary clutch disk and input cogwheel; \( J_2 \) is the equivalent secondary shaft inertia, including output gearbox shaft, differential gear and axle shaft, while \( J_w \) is the equivalent inertia of wheels and vehicle mass. Equivalent elastic stiffness \( k_{2w} \) of the secondary shaft takes into account both output gearbox shaft, axle shaft and tire elasticity, viscous friction coefficients of the main and secondary shaft are \( b_1 \) and \( b_{2w} \) respectively.

**Remark 3.1:** It is worth noting that in (1) engine crankshaft and main shaft have been modelled as pure inertial elements, and their elasticity have been neglected with respect to the secondary shaft spring. Frequency-domain analysis, not reported here for the sake of space, supports...
this assumption for the examined driveline.

In the following, the discrete dynamics and definitions of $T_c$ and $T_{gm}, T_{gs}, r$ are reported.

A. Clutch model

Clutch torque is modelled as stick-slip friction torque between engine and main shaft, with maximum amplitude given by the continuous control input $T_{cM} \geq 0$.

Remark 3.2: From a physical viewpoint, $T_{cM}$, which represents the maximum transmissible clutch torque, is a function of the clutch actuator position through the transmissibility curve. It is assumed to be a smooth signal with bounded 1st and 2nd time derivatives.

A graphical representation of the automaton modelling the clutch dynamics is shown in Fig. 2. The set of discrete states for locked and slipping clutch disks is

$$Q_c = \{c_{lock}, c_{slip}\},$$

with invariant sets

$$I(c_{lock}) = \{x : \omega_c = \omega_1\}, \quad I(c_{slip}) = \mathbb{R}^5.$$ The set of edges is

$$E_c = \{(c_{lock}, c_{slip}), (c_{slip}, c_{lock})\}.$$ Imposing $\omega_c = \omega_1$ and $\dot{\omega}_c = \omega_1$ in (1), the torque to be transmitted by the clutch in order to maintain the engine and the main shafts at the same speed (locked clutch) is given by

$$T_c^* = J_1T_c + J_2(T_{gm} + b_1\omega_1).$$ (2)

According to stick-slip friction model, clutch torque $T_c$ is expressed with dependence on discrete state $q_c$ as

$$T_c = T_c^* \quad \text{if} \quad q_c = c_{lock}$$

$$T_c = T_{cM}\text{sgn}(\omega_c - \omega_1) \quad \text{if} \quad q_c = c_{slip}.$$ (3)

Guard conditions are defined as a function of $T_c^*$, control input $T_{cM}$ and speeds $\omega_c, \omega_1$

$$G(c_{lock}, c_{slip}) = \{(T_c^*) \geq T_{cM}\}$$

$$G(c_{slip}, c_{lock}) = \{(T_c^*) \leq T_{cM}\} \text{ and } (\omega_c = \omega_1).$$

No discrete-time signals and reset map are defined for the clutch automaton.

Remark 3.3: It is worth noting that when $q_c = c_{lock}$ the invariant set condition $\omega_c = \omega_1$ is always satisfied if $|T_c| = |T_c^*| \leq T_{cM}$, i.e. clutch disks remain locked if the transmitted torque is lower or equal to the maximum transmissible one, otherwise they start slipping.

During condition $q_c = c_{slip}$, it is assumed or that clutch disks are slipping, or that clutch is completely opened, whereby $T_{cM} = 0$.

B. Gearbox model

Gearbox discrete dynamics is introduced in order to model the engaged, disengagement, neutral and synchronization phases, and to describe speed selection. Considering a 6-speed gearbox, a continuous-time discrete-valued control input $g^* \in \{0, 1, 2, 3, 4, 5, 6\}$ representing the requested gear is defined with the following meaning. When $g^* \neq 0$, engagement of gear $g^*$ is requested, while $g^* = 0$ stands for neutral phase request.

Remark 3.4: From a physically meaningful viewpoint, control input $g^*$ is an abstraction of the operations performed by the gearbox actuator.

Two discrete state variables are defined as $q_g \in Q_g$, $q_r \in Q_r$, with

$$Q_g = \{g_{eng}, g_{neutr}, g_{sync}\}, \quad Q_r = \{0, 1, 2, 3, 4, 5, 6\}.$$ A graphical representation of the automaton for the variable $q_g$ and the finite state machine for the discrete state $q_r$ are depicted in Figs. 3 and 4.

Discrete state $q_g = g_{eng}$ represents the engaged phase, when main and secondary shafts are locked by the gear ratio $r$, or when gear disengagement ($g^* = 0$) has been requested but not executed yet; $q_g = g_{neutr}$ represents the neutral phase, when gearbox shafts are decoupled; $q_g = g_{sync}$ represents the synchronization phase, lasting from the gear engaging request $g^* \neq 0$ to the speed synchronization $\omega_1 = r\omega_2$. In Fig. 4, $q_r \in \{1, \ldots, 6\}$ represents the gear which is engaged or is about to be synchronized, while $q_r = 0$ holds in neutral phase.

According to graphs in Figs. 3 and 4, the sets of edges of the discrete states $q_g, q_r$ are defined as

$$E_g = \{(g_{eng}, g_{neutr}), (g_{neutr}, g_{sync})\},$$

$$E_r = \{(g_{sync}, g_{eng}), (g_{sync}, g_{neutr})\}.$$

\[\text{Fig. 2. Clutch automaton (discrete state } q_c).\]

\[\text{Fig. 3. Gearbox automaton (discrete state } q_g).\]

\[\text{Fig. 4. Gear ratio finite state machine (discrete state } q_r).\]
Supposing that the gearbox is engaged with gear ratio \( r \), imposing \( \omega_1 = r \omega_2 \) and \( \dot{\omega}_1 = r \dot{\omega}_2 \) in (1), the torque to be transmitted by the input gearbox shaft in order to maintain the main and secondary shafts at the same speed can be expressed by

\[
T_g^* = \frac{J_2(T_c - b_1\omega_1) + rJ_1T_{2w}}{J_2 + r^2J_1}.
\]

Transitions of the gearbox automaton in Fig. 3 are enabled according to the following guards depending both on control input \( g^* \), on state-dependent signals \( T_g, r \) and on speeds \( \omega_1, \omega_2 \):

\[
G_g(g_{\text{eng}}, g_{\text{neutr}}) = \{ (g^* = 0) \text{ and } (|T_g| \leq T_{\text{diseng}}) \}
\]

\[
G_g(g_{\text{neutr}}, g_{\text{sync}}) = \{ g^* \neq 0 \}
\]

\[
G_g(g_{\text{sync}}, g_{\text{eng}}) = \{ \omega_1 = r \omega_2 \}
\]

\[
G_g(g_{\text{sync}}, g_{\text{neutr}}) = \{ g^* = 0 \}
\]

where \( T_{\text{diseng}} \geq 0 \) is an input representing the threshold torque allowing gear disengagement.

Remark 3.5: First three edges in the definition (4) of \( E_q \) correspond to the correct sequence for gear shifting, i.e. the only way to perform a gear ratio changing is passing from the neutral phase. It is worth noting that transition to neutral state is enabled only if the transmitted gearbox torque is lower than the threshold \( T_{\text{diseng}} \). Last edge represents the stopping request of the synchronization phase.

Synchronization of automata for discrete variables \( q_g \) and \( q_r \) is performed by means of an internal discrete-time signal. It is defined as output variable associated to the edge \((q_{\text{eng}}, q_{\text{neutr}})\) as:

\[
\sigma_c(q_{\text{eng}}, q_{\text{neutr}}) = \text{EngToNeutr}
\]

and it is an input discrete-time signal in the finite state machine for \( q_r \). Guards for \( q_r \) are defined as

\[
G_r(i, 0) = \{ \sigma_c = \text{EngToNeutr} \}
\]

\[
G_r(0, i) = \{ g^* = i \}, \quad \forall \ i.
\]

Remark 3.6: Discrete event \( \sigma_c = \text{EngToNeutr} \) is used to define a state-independent guard condition, which enables the transition from \( q_r = i \) to \( q_r = 0 \), \( \forall \ i \), when the transition from \( g_{\text{eng}} \) to \( g_{\text{neutr}} \) occurs. Hence, it follows that when \( q_g = g_{\text{neutr}} \) it holds \( q_r = 0 \), while when \( q_r \neq 0 \), it holds \( q_g = g_{\text{sync}} \) or \( q_g = g_{\text{eng}} \).

Discrete-valued variable \( r \) represents the map of gear ratios and is defined depending only on \( q_g \) as \( r = r(q_r), q_r \neq 0 \). It is assumed that \( r(0) = 0 \). Continuous-time variables \( T_{g_m}, T_{g_s} \) are defined with dependance on \( q_g \) as

\[
T_{g_m} = T_g^*, \quad T_{g_s} = rT_g^* \quad \text{if} \quad q_g = g_{\text{eng}}
\]

\[
T_{g_m} = T_g = 0 \quad \text{if} \quad q_g = g_{\text{neutr}}
\]

\[
T_{g_m} = T_{g_s} = T_{\text{sync}} \text{sgn}(\omega_1 - r \omega_2) \quad \text{if} \quad q_g = g_{\text{sync}}.
\]

where control input \( T_{\text{sync}} \geq 0 \) is the Coulomb friction torque applied by the synchronizer.

The invariant set for the state \( g_{\text{eng}} \) is given by:

\[
I(g_{\text{eng}}) = \{ x : \omega_1 \equiv r \omega_2 \}
\]

which is the condition imposed by the engagement of gear with ratio \( r \), while other invariant sets are the whole state space. No reset assignment is needed.

IV. MODEL VALIDATION AND EXPERIMENTAL RESULTS

Validation of the hybrid model of the driveline presented in Section III has been performed on experimental data relative to a 1st to 2nd gear shift. Experiments have been realized on a testbench constituted by a commercial sport car with transaxle layout equipped with hydraulically servo-actuated clutch and gearbox. An Autobox dSpace board has been interfaced with the Electronic Control Unit (ECU) for data acquisition.

In the experimental set-up, the measured variables, sampled every 10 ms, are: engine speed \( \omega_e \), gearbox input shaft speed \( \omega_1 \) and wheel speed \( \omega_2 \). Torque \( T_e \) is the estimated engine torque obtained by the ECU. The load torque \( T_w \) applied to the wheels is supposed to be constant and known. The maximum transmissible clutch torque \( T_{c,M} \) is reconstructed from the measured clutch actuator position through the nominal torque transmissibility curve. In order to define the discrete-valued gearbox input signal \( g^* \), a function mapping the measured selection and engagement actuator positions into \( g^* \) is implemented. This function is based on thresholds for the gearbox actuator position which define the gearbox operation.

A Matlab driveline model has been developed exploiting Simulink/Stateflow toolboxes. Model parameters have been identified both by means of physical laws and by a numerical parameter estimation method based on nonlinear least square optimization, comparing measured and simulated outputs. Model parameters are reported in Table I. In Fig. 5 the experimental and simulation results relative to the gear shift are compared; control inputs, speed profiles from experiment and simulation and discrete states of the gearbox and clutch automata are shown. While applying constant 250 Nm engine torque, with 20 Nm wheel torque, the gear shift request from 1st to 2nd gear is executed. Engine torque goes to −90 Nm at \( t = 0.13 \) s, engine speed decreases and the clutch is opened. After the disengagement phase (from 0.145 s to 0.18 s), the gearbox is neutral. When \( g^* = 2 \), the 2nd gear synchronization phase is executed from 0.23 s to 0.27 s, when primary and secondary gearbox speeds reach the same value. After the 2nd gear is engaged, engine torque is increased up to the starting value and the clutch is smoothly locked, in order to have smooth speed profiles. At \( t = 0.49 \) s the clutch is locked \( (\omega_e = \omega_1) \).

Comparison of Figs.5 (b) and (c) shows that the proposed model reproduces the main phenomena in the driveline during gear shifting, both in terms of timing and speed transients. The simulation model accurately reproduces

\[
E_r = \bigcup_{i=1}^{6} \{ (i, 0), (0, i) \}.
\]
engine and wheel speeds, which have a relatively low harmonic content. The model is able to capture driveline damped oscillations in the $\omega_1$ speed, which are due to complex eigenvalues of the mass-spring-damper systems. Nevertheless, real and simulated transients differ during clutch locking phase (from 0.35 s). This is mainly due to gearbox backlash, clutch damper effect and other neglected nonlinearities in the driveline shaft, and to high model sensitivity with respect to synchronizer torque and clutch transmissibility curve. It is worth noting that, despite of the long main shaft between engine and gearbox due to the transaxle layout, the proposed model, which contains only one damper-spring dynamics of the secondary/axle shaft, is able to accurately model the transients.

V. CONCLUSIONS

A hybrid system modelling approach encompassing continuous-time signals and discrete events has been presented. In this framework, modelling of a vehicle driveline for control design purpose has been proposed. It takes into account the fundamental hybrid properties of the mechanical dynamics and clutch and gearbox actuators. Experimental results confirm that the proposed model is sufficiently accurate to describe gear shift. The subject of future research activities will be the design of controllers for automated gear shift, exploiting the proposed hybrid system framework.

### REFERENCES


### TABLE I

<table>
<thead>
<tr>
<th>Driveline model parameters</th>
<th>Value</th>
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<tbody>
<tr>
<td>$J_e$</td>
<td>0.184 kgm²</td>
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<tr>
<td>$J_1$</td>
<td>0.0672 kgm²</td>
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<tr>
<td>$b_1$</td>
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<td>$J_2$</td>
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<td>$k_{2w}$</td>
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<td>$b_{2w}$</td>
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<tr>
<td>$J_w$</td>
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</tr>
<tr>
<td>$(r_1, r_2)$</td>
<td>(3.29, 2.16)</td>
</tr>
<tr>
<td>$T_{diseng}$</td>
<td>5 Nm</td>
</tr>
<tr>
<td>$T_{sync}$</td>
<td>100 Nm</td>
</tr>
</tbody>
</table>

Fig. 5. Experimental and simulation results. (a) Torque $T_e$ (black), $T_{cM}$ (blue), $T_w$ (red). (b, c) Speed $\omega_e$ (black), $\omega_1$ (blue), $r_2 - \omega_w$ (red) from experiments (b) and simulation (c). (d) Gearbox control input $g^*$ (1 - 1st gear, 0- neutral, 2 - 2nd gear). (e) Gearbox discrete states $q_1$ (black), $q_2$ (blue) (1 - $g_{eng}$, 0 - $g_{neutr}$, 3 - $g_{sync}$). (f) Clutch discrete state $q_c$ (1 - $e_{lock}$, 0 - $e_{slip}$).