Implicit fault-tolerant control: application to induction motors

Claudio Bonivento\textsuperscript{a}, Alberto Isidori\textsuperscript{a,b,c}, Lorenzo Marconi\textsuperscript{a,*}, Andrea Paoli\textsuperscript{a}

\textsuperscript{a}Center for Research on Complex Automated Systems (CASY) “Giuseppe Evangelisti”, DEIS, Department of Electronic, Computer Science and Systems, University of Bologna, Via Risorgimento 2, 40136 Bologna, Italy
\textsuperscript{b}Dipartimento di Informatica e Sistemistica, University of Rome, Italy
\textsuperscript{c}Department of Systems Science and Mathematics, Washington University, St. Louis, MO 63130, USA

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Abstract

In this paper we propose an innovative way of dealing with the design of fault-tolerant control systems. We show how the nonlinear output regulation theory can be successfully adopted in order to design a regulator able to offset the effect of all possible faults which can occur and, in doing so, also to detect and isolate the occurred fault. The regulator is designed by embedding the (possible nonlinear) internal model of the fault. This idea is applied to the design of a fault-tolerant controller for induction motors in presence of both rotor and stator mechanical faults.

Keywords: Fault-tolerant control; Induction motor; Output-feedback control; Internal model control; Adaptive control

1. Introduction

Many efforts in the control community have been recently devoted to study “fault-tolerant” control (FTC) systems, namely: control systems able to detect incipient faults in sensors and/or actuators on the one hand and on the other, to promptly adapt the control law in such a way as to preserve pre-specified performances in terms of quality of the production, safety, etc.

The most common approach in dealing with such a problem (see Frank, 1990 and the reference therein) is to split the overall design in two distinct phases. The first phase addresses the so-called “fault detection and isolation” (FDI) problem, which consists of designing a dynamical system (filter) which, by processing input/output data, is able to detect the presence of an incipient fault and to isolate it from other faults and/or disturbances. Once the FDI filter has been designed, the second phase usually consists of designing a supervisory unit which, on the basis of the information provided by the FDI filter, reconfigures the control so as to compensate for the effect of the fault and to fulfill performances constraint. In general, the latter phase is carried out by means of a parameterized controller which is suitably updated by the supervisory unit.

It is clear from this description that the classical approach to FDI and FTC relies upon a “certainty equivalence” idea extensively used in the context of adaptive control, since it is based on the explicit estimation of unknown time varying signals/parameters (in the specific case the faults) by the FDI filter and the subsequent explicit reconfiguration of the controller in presence of faults.

The aim of this paper is to follow a different approach to FTC. Specifically, we address the case in which the faults affecting the controlled system can be modeled as functions (of time) within a finitely parametrized family. Then, we design a controller which embeds an internal model of this family, whose purpose is to generate a supplementary control action which compensate for the presence of any of such faults,
regardless their entity. In other words, the control reconfiguration does not rely upon an explicit FDI design but, indeed, is achieved by a proper design of a dynamic controller which is implicitly fault tolerant to all the possible faults whose model is embedded in the regulator. This idea will be pursued using the theoretical machinery of the (non-linear) output regulation theory (see Byrnes, Delli Priscoli, & Isidori, 1997a; Byrnes, Delli Priscoli, Isidori, & Kang, 1997b) under the assumption that the side-effects generated by the occurrence of the fault can be modeled as an exogenous signal generated by an autonomous “steadily” system (the so-called “exosystem”). As opposite to certainty equivalence-based adaptive control, the distinctive feature of output regulation theory relies upon designing a controller which, by embedding an internal model of exosystem, is able to offset the effect of any “exosystem-generated” signal without requiring an explicit estimation of such signal. It is interesting to see that, in this framework, the FDI phase, which is usually the starting point in the design of a FTC systems, is postponed to that of control reconfiguration since it can be carried out by testing the state of the internal model unit which automatically activates to offset the presence of the fault. In this paper the approach outlined above is specialized to the design of a FTC system for induction motors (IM). As all the magnetic rotating machines, the IM is subjected to rotor and stator failures caused by a combination of thermal, electrical, mechanical, magnetic and environmental stresses. Due to these stresses the IM can operate into a failure condition whose effects show with spurious harmonic currents arising in the stator circuit (see Bellini, Filippetti, Franceschini, & Tassoni, 2000; Vas, 1994; Benbouzid, 1997) with frequencies which are directly related to the kind of the fault (in general stator or rotor fault) and amplitude and phase which depend on the severity of the fault.

This allows us to see the fault-tolerant problem in the perspective outlined-above and to cast the problem as an output regulation problem. More in detail, we show how an indirect field-oriented (IFO) controller (see Ortega, Nicklasson, & Espinosa, 1996; Peresada & Tonielli, 2000) and the reference therein) can be “augmented” with a dynamic unit designed so as to compensate the unknown spurious harmonic currents arising in the stator circuit in presence of rotor or stator faults. In this way, a controller which is implicitly fault tolerant to all the faults belonging to the model embedded in the regulator is obtained.

Notations. The notations used throughout the paper are rather standard. The vector norm of \( x \in \mathbb{R}^n \) is simply denoted by \( \| x \| \). If \( f \) is a function in \( L^2_{\infty} \), the set of all piecewise-continuous functions \( f : [0, \infty) \rightarrow \mathbb{R}^k \) which are bounded, the norms \( \| f \|_{\infty} \) and \( \| f \|_a \) denote, respectively, \( \| f \|_{\infty} = \sup_{t \in [0, \infty)} \| f(t) \| \) and \( \| f \|_a = \lim_{t \rightarrow \infty} \| f(t) \| \). By blockdiagonal matrices \( A_1, \ldots, A_k \) we denote the block diagonal matrix with the \( k \) matrices \( A_i \) on the diagonal, while \( \text{skew}(r) \), \( r \in \mathbb{R} \), denotes the skew symmetric \( 2 \times 2 \) matrix with \( r \) on the upper diagonal and \( -r \) on the lower diagonal.

2. The induction motor model and the indirect field-oriented controller

2.1. The induction motor model

In this section we briefly review the model of the IM. For a more exhaustive treatment on how this model can be derived the interested reader can refer to Novotny and Lipo (1998).

Under assumptions of linear magnetic circuits and balanced operating conditions, the equivalent two-phase model of the symmetrical IM, represented in an arbitrarily rotating two-phase reference frame \( (d-q) \), is (see Marino, Peresada, & Valigi, 1993)

\[
\dot{x} = f(x, \omega_0) + B(u + V) + dT_L, \\
y = Cx, \\
\dot{\omega}_0 = \omega_0, \quad \omega_0(0) = 0
\]

in which \( x = (\omega, \Psi_d, \Psi_q, i_d, i_q)^T \) is the state vector, \( u = (u_d, u_q)^T \) is a control vector and \( y = (\omega, i_d, i_q)^T \) is a vector of measurable variables. The state variables are defined as follows: \( \omega \) is the rotor speed, \( (\Psi_d, \Psi_q) \) are the components of the rotor flux, \( (i_d, i_q) \) are the components of the stator current vector. The input \( T_L \) is the unknown load torque and \( V = (V_d, V_q)^T \) represents an exogenous input which is zero in case the IM works in un-faulty mode while is a bounded (unknown) signal in the presence of faults (see the treatment in Section 3.1). The variable \( \omega_0 \) represents the angular position of the rotating \( (d-q) \) reference frame with respect to the \( a \)-axis of the fixed stator reference frame \( (a-b) \), and its rate of change \( \omega_0 \) is viewed as an additional control. The relation between the original \( (a-b) \) and transformed \( (d-q) \) variables is given by

\[
x_{dq} = e^{-j\omega_0} x_{ab} \quad \text{where} \quad e^{-j\omega_0} = \begin{bmatrix} \cos \omega_0 & \sin \omega_0 \\ -\sin \omega_0 & \cos \omega_0 \end{bmatrix}, \\
x_{ab} = e^{j\omega_0} x_{dq}
\]

where \( x_{ab} \) stands for two-dimensional vectors in the \( (y-z) \) reference frame. The vector-valued function \( f(x, \omega_0) \) and the constant matrices \( B \) and \( d \) are, respectively,

\[
f(x, \omega_0) = \begin{pmatrix}
\mu(\Psi_d i_d - \Psi_q i_q) \\
-2\Psi_d (\omega_0 - \omega) \Psi_d + 2Lmi_d \\
-(\omega_0 - \omega) \Psi_d - 2\Psi_q + 2Lmi_q \\
z\beta \Psi_d + \beta \omega \Psi_q - \gamma i_d + \omega_0 i_q \\
-\beta \omega \Psi_d + \beta \omega \Psi_q - \omega_0 i_d - \gamma i_q
\end{pmatrix},
\]

\[
B = \frac{1}{\sigma} \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}^T, \quad d = \begin{pmatrix} -1/\sigma & 0 & 0 & 0 & 0 \end{pmatrix}^T.
\]
where the positive constants in the model are related to the electrical and mechanical parameters of IM as follows:

\[
\begin{align*}
\sigma &= L_s \left(1 - \frac{L_m^2}{L_d L_q}\right), \\
\beta &= \frac{L_m}{\sigma L_s}, \\
\mu &= \frac{3}{2} \frac{L_m}{J L_r}, \\
\varepsilon &= \frac{R_s}{L_r}, \\
\gamma &= \left(\frac{R_s}{\sigma} + \frac{\sigma L_m}{\sigma L_s}\right)
\end{align*}
\]

with \( J \) the rotor inertia, \( R_s, R_t, L_s, L_r \) the stator/rotor resistances and inductances, respectively, \( L_m \) the magnetizing inductance.

2.2. Control objectives

General specifications for speed-controlled electric drives consider two outputs: rotor speed \( \omega \) and rotor flux amplitude \(|\Psi| = \sqrt{\Psi_d^2 + \Psi_q^2}\). These variables are supposed to track two reference signals denoted, respectively, by \( \omega^* \) and \( \Psi^* \), which are assumed to be smooth functions of time. A further control objective consists of having \( \Psi_d(t) \) asymptotically decaying to zero, a property known as steady-state flux decoupling. Hence, given \( \omega^* \) and \( \Psi^* \), the problem consists of designing a dynamic output feedback controller of the form

\[
\begin{align*}
\dot{v} &= \alpha(v, y, \omega^*, \Psi^*), \\
u &= \beta(v, y, \omega^*, \Psi^*), \\
o_0 &= \pi(v, y, \omega^*, \Psi^*)
\end{align*}
\]

such that for all initial states \( x(0) \in \mathbb{R}^5 \) and for all possible constant torque load \( T_L \), the trajectories of the closed-loop system are bounded and

\[
\lim_{t \to \infty} |\omega(t) - \omega^*(t)| = 0, \\
\lim_{t \to \infty} |\Psi_d(t) - \Psi^*(t)| = 0, \\
\lim_{t \to \infty} \Psi_q(t) = 0.
\]

2.3. A global indirect field-oriented controller

The FTC scheme proposed in this paper is based on a parallel structure in which a “safe controller”, namely a controller able to meet the control specifications in absence of faults, is completed with a fault tolerance unit able to provide the extra control effort needed to compensate for the fault effect. The attempt in this paper is to identify a design procedure such that the structure of the safe controller and its tuning is not influenced by the design of the fault tolerance unit. In other words the latter should be, ideally, a sort of plug and play device to be inserted in an already existing safe control loop in order to enhance the fault tolerance features. Along these lines, the goal of this section is to briefly present the structure of a possible safe controller and to highlight the main features which are needed in order to carry out the design of the fault tolerance unit which will be presented in the next section. To this end we specialize our analysis to the controller proposed in Peresada and Tonielli (2000) which belongs to the family of the so-called indirect field oriented controllers (see Novotny & Lipo, 1998), usually adopted in case the flux is not available for measurement. For this controller we highlight specific properties, not discussed in Peresada and Tonielli (2000), which are instrumental for the analysis presented in Section 3.

The idea in Peresada and Tonielli (2000) is to design the control inputs \((u_d, u_q)\) and \(o_0\) in order to force the overall system to behave as the cascade connection of two subsystems, called the flux subsystem and the speed subsystem.

To begin the analysis of the flux subsystem, define error variables as

\[
\begin{align*}
\tilde{\Psi}_d &= \Psi_d - \Psi^*, \\
\tilde{\Psi}_q &= \Psi_q
\end{align*}
\]

and set

\[
\begin{align*}
i_d^* &= i_d - i_d^* \quad \text{where } i_d^* := \frac{1}{\alpha L_m} (\varepsilon \Psi^* + \tilde{\Psi}_d). \\
\end{align*}
\]

Trivial computations show that, choosing the control input \( u_d \) and the rate of the \( (d - q) \) reference frame \( o_0 \) as

\[
\begin{align*}
u_d &= \alpha(-K_d i_d + \gamma_i d - o_0 \Psi_q - \varepsilon \Psi^* + i_d^*) + \mu \tilde{\Psi}_d \\
&:= \ddot{u}_d + \sigma \tilde{\Psi}_d,
\end{align*}
\]

\[
o_0 = \omega + \frac{1}{\Psi^*} (2L_m i_q + \beta o i_d^*)
\]

in which \( K_d \) is a design parameter and \( \tilde{u}_d^f \) an additional control input (which will be spent for fault compensation), the dynamics of the state variables \((\Psi_d, \Psi_q, i_d)\), in the new coordinates \( (2)–(3) \), are described by the following equations (which define the flux subsystem):

\[
\begin{align*}
\dot{\tilde{\Psi}}_d &= -2 \tilde{\Psi}_d + s_o \tilde{\Psi}_q + 2L_m \dot{i}_d, \\
\dot{\tilde{\Psi}}_q &= -s_o \tilde{\Psi}_d - \tilde{\Psi}_q - \beta o i_d^*, \\
\dot{i}_d &= -2 s_o \tilde{\Psi}_d + \beta o \tilde{\Psi}_q - K_d i_d + v_d
\end{align*}
\]

in which \( s_o := o_0 - \omega \) is the so-called slip angular frequency and \( v_d := \tilde{u}_d^f + V_d \) is viewed as an exogenous input. These dynamics, having set

\[
\begin{align*}
x_{\text{f}} := (\tilde{\Psi}_d, \tilde{\Psi}_q, \tilde{i}_d)^T
\end{align*}
\]

can be rewritten in more compact form as

\[
\begin{align*}
x_{\text{f}} &= A_{\text{f}}(\omega, s_o)x_{\text{f}} + B_{\text{f}} v_d, \\
\end{align*}
\]

where \( A_{\text{f}}(\omega, s_o) \) and \( B_{\text{f}} \) are suitably defined matrices with the entries of the matrix \( A_{\text{f}} \) which depends on the variables \( \omega \) and \( s_o \) in a skew symmetric way.

We now take into account the remaining state variables of (1) to define the speed subsystem. To this end, define the additional error variable \( \tilde{\omega} := \omega - \omega^* \), and set

\[
\begin{align*}
i_q^* &= \frac{1}{\mu \Psi^*} (-K_o \tilde{\omega} + \tilde{T}_L + \tilde{\omega}^*).
\end{align*}
\]

In the expression of \( i_q^* \) above, \( K_o \) is a positive constant and \( \tilde{T}_L \) is an auxiliary state variable of the controller, introduced to offset the unknown load torque \( T_L \), whose dynamics will be specified later. Moreover, let \( i_{q1}^* \) denote the part
of the derivative of \( i_q^* \) depending on the measurable variables \((i_d, i_q, \omega)\) given by

\[
i^*_{q1} := \frac{1}{\omega^*} \left[ K_o (K_o \omega - \mu \Psi^* i^*_{q}) + \Psi^* + \omega^* \right] - \frac{\Psi^*}{\omega^*} i^*_{q}.
\]

With this notation in mind, the control law for the input \( u_q \) is designed, in Peresada and Tonielli (2000), in the following way:

\[
u_q = \sigma [ (K_q - \gamma) \tilde{i}_q + \omega_o i_d + \beta_0 \omega^* + i^*_{q1} - \frac{1}{\omega^*} (\Psi^* i^*_{q} - K_\xi \tilde{\xi}) ] + \sigma u^e_q := \bar{u}_q + \sigma u^e_q,
\]

\[
\hat{\dot{\xi}} = K_q \Psi_i q,
\]

\[
\hat{T}_L = -K_T \dot{\omega}_o,
\]

where \( K_q \) and \( K_\xi \) are design parameters, \( K_q \) and \( K_T \) are fixed (though arbitrary) positive constants and \( u^e_q \) is a new control input, used for fault compensation, to be fixed later. The subsystem thus obtained (which defines the speed subsystem), having set

\[
\eta := \Psi^* \tilde{i}_q,
\]

\[
\hat{T}_L := \hat{T}_L - T_L
\]

is described by equations of the form

\[
x = A x + A_d(x, t) x + B_d(x, t) v + B v_q
\]

in which \( x := (\hat{T}_L \omega \tilde{\xi} \eta)^T \) and

\[
A_s = \begin{pmatrix}
0 & K_T & 0 & 0 \\
-1 & -K_o & 0 & \mu \\
0 & 0 & 0 & K_q \\
\frac{K_o}{\mu} & 0 & -K_\xi & -K_q
\end{pmatrix},
\]

\[
A_d(x, t) = \begin{pmatrix}
0 & 0 & 0 & 0 \\
-1 & -K_o & 0 & \mu \\
0 & 0 & 0 & 0 \\
\frac{K_o}{\mu} & a_{42} & 0 & K_o
\end{pmatrix},
\]

\[
B_s(x, t) = \begin{pmatrix}
b_{21} & 0 & b_{41} \\
b_{22} & 0 & b_{42}
\end{pmatrix}^T, \quad B_v = (0 0 0 1)^T
\]

with

\[
a_{42} := -\beta \Psi^* - \frac{K_\xi}{\mu}, \quad b_{21} := \frac{1}{\Psi^*} (T_L + \omega^*),
\]

\[
b_{22} := -\mu (\dot{i}_d + i^*_{q1}), \quad b_{41} := -\beta \omega^* + \frac{K_\xi}{\Psi^*} (T_L + \omega^*),
\]

\[
b_{42} := \beta \Psi^* - K_o i_d - K_o \dot{i}_d.
\]

As for \( \dot{v}_d \) in the case of the flux subsystem, here \( \dot{v}_q := \dot{v}_q^e + \dot{v}_q \) is viewed as an exogenous input.

The stability analysis of the overall system can be conveniently done by considering systems (6) and (8) as a cascade connection, with the flux subsystem (6), with input \( v_d \), feeding the speed subsystem (8), with inputs \( v_q \) and \( x_r \).

The intuition behind this is given by the fact that the feedback from the speed subsystem to the flux subsystem, coming from the fact that \( \dot{A}_k(\omega, s_o) \) depends on speed variables, does not play any role in the stability analysis as \( \dot{A}_k(\omega, s_o) \) turns out to be skew symmetric with respect to \( \omega \) and \( s_o \).

On this intuition is based the proof of the next proposition, reported for convenience in Appendix A which states that a sufficiently large value of \( K_d \) and \( K_q \) and \( K_\xi \) renders the overall system (5)–(8) input-to-state stable with respect to the inputs \((v_d, v_q)\) with arbitrarily large restrictions on the inputs and arbitrarily small linear gains.

**Proposition 1.** Let \( M \) be an arbitrary positive number and set \( K_d = k K_d \), \( K_q = k K_q \), \( K_\xi = k K_\xi \) where \( K_d \), \( K_q \) and \( K_\xi \) are fixed positive numbers. Then there exists \( k^* > 0 \) such that for all \( k \geq k^* \) the overall system (5)–(8) is input-to-state stable without restriction on the initial state, restrictions \( (M, \infty) \) on the inputs \((v_d, v_q)\) and linear gain functions. In particular there exist numbers \( \gamma > 0 \) and \( \gamma_0 > 0 \) such that for each \( x^0 \in \mathbb{R}^7 \) and each measurable \((v_d, v_q)\) such that \( \|v_d\|_{\infty} \leq M \), the solution of (5)–(8) with \( x(0) = (x_i(0), x_0(0)) = x^0 \) exists for all \( t \geq 0 \) and satisfies

\[
\|x\|_{\infty} \leq \max \left\{ \frac{\gamma_0}{k} \|x^0\|, \frac{\gamma}{k} \max \left\{ \|v_d\|_{\infty}, \|v_q\|_{\infty} \right\} \right\},
\]

\[
\|x\|_{\infty} \leq \frac{\gamma}{k} \max \left\{ \|v_d\|_{\infty}, \|v_q\|_{\infty} \right\}.
\]

The previous result implies that, in case \( v_d \) and \( v_q \) are identically zero, the overall closed-loop system given by (5)–(8) is globally asymptotically stable namely the control objectives specified in Section 2.2 are achieved for every initial state \( x(0) \) of the IM (1). Hence the controller presented above represents a possible “safe controller” in the sense that it is able to achieve the prescribed closed-loop properties in case no faults happen on the system. In addition the previous result claims that the IFO controller is “robust” with respect to the matched signals \((v_d, v_q)\), as it achieves input-to-state stability with a linear gain proportional to \( 1/k \). This property will play a crucial role in the next section where the fault-tolerant unit able to offset the effect of possible faults is presented.

We conclude this section by just remarking how the design of the fault-tolerant unit carried out in the next section does not necessarily rely on the specific structure of the IFO controller presented in Peresada and Tonielli (2000) and briefly recalled above. Indeed the only key feature required to the “safe controller” in order to design the fault-tolerant unit, is to achieve the control objectives robustly, in the sense of the previous proposition, with respect to exogenous signals matched with the control inputs. Any other controller,
indirect field oriented or not, able to achieve for the error system input-to-state stability with arbitrarily large restrictions on matched signals and whose linear gain can be arbitrarily set are all admissible and potentially fits in the framework proposed in this paper.

3. The implicit fault-tolerant controller

3.1. Fault scenario and faulty model

In this section we briefly review how the model of the IM modifies in presence of faults. The fault scenario considered in this paper addresses mechanical faults caused both by rotor and stator failures of the IM. Several research efforts have focused in the last years on the study of the effect of faults in the IM, see Bellini et al. (2000), Gentile et al. (1990), Gentile, Rotondale, Martelli, and Tassoni (1994), Vas (1994), Williamson and Smith (1982), and Williamson and Mirzoian (1985). All these works, mainly developed within the domain of electrical engineering, are based on approximate mathematical description of the faulty motor and on appropriate experimental validation. As a matter of fact the presence of faults in general introduces asymmetries in the motor circuit which make a precise and rigorous modeling an impossible task.

With reference to Vas (1994), the faults dealt with in this paper can be summarized in the following two classes:

- rotor asymmetries, mainly due to broken bars or dynamic eccentricity;
- stator asymmetries, mainly due to static eccentricity.

Following the theory in Vas (1994), it turns out that the presence of stator and rotor faults generates asymmetries in the IM, yielding some slot harmonics in the stator winding. In the (a–b) reference frame, it is possible to model this effect by adding a sinusoidal corruption term to the stator currents values. Specifically, letting $i_{af}^s(t)$ and $i_{bf}^s(t)$ denote the values of the stator current in the absence of faults and $i_{af}^s(t)$ and $i_{bf}^s(t)$ the corresponding values in the presence of faults, the latter can be expressed in the form

$$i_{af}^s(t) = i_{af}^s(t) + A \sin(\omega_c(t) + \phi),$$

$$i_{bf}^s(t) = i_{bf}^s(t) + A \cos(\omega_c(t) + \phi)$$

in which

$$\omega_c(t) = 2\pi \int_0^t f_e(\tau) d\tau,$$

(11)

where $f_e(t)$ is a function which depends on the specific fault. For example, faults caused by rotor asymmetries (typically due to broken bars or dynamic eccentricity) yield harmonic components at frequencies

$$f_e = f_{rbb} = \left(1 \pm 2k \frac{s_\omega}{\omega_0}\right) f$$

(13)
in which (see Section 2) $s_\omega = \omega_0 - \omega$ is the slip angular frequency, $f$ the supply frequency and $k$ a positive integer. On the other hand, faults generated by stator asymmetries (typically induced by static eccentricity) generate harmonic components at the frequency

$$f_e = f_{ssc} = f.$$ (14)

As far as the amplitude $A$ and the phase $\phi$ in (11) are concerned, they depend on the entity of the rotor or stator asymmetry and then cannot be considered known since depend on the specific fault severity.

Similarly, once the variables are expressed in the $(d-q)$ reference frame, it turns out that the stator currents in presence of (stator or rotor) asymmetries change into

$$i_d = i_{af}^d + A \sin(\omega_c(t) + \omega_0(t) + \phi),$$

$$i_q = i_{bf}^d + A \cos(\omega_c(t) + \omega_0(t) + \phi),$$

(15)

where $\omega_0$, introduced in the previous section, denotes the angular position of the $(d-q)$ reference frame. Few assumptions are made in the following in order to simplify relation (15). First of all, for the sake of simplicity, we concentrate on the case in which the possible frequencies which characterize the sinusoidal additive terms in (15) are constants. This corresponds to assume that the following two hypotheses hold:

(a) the reference angular velocity $\omega^*$ is constant and a possible fault is allowed to arise only when the steady state has been reached;

(b) the frequencies which characterize the faulty current are “frozen” to the steady state values, namely to the values obtained when the state of the system assumes the steady state value. This is equivalent to assume that the frequencies are not dependent on the actual state of the plant but only on the reference to be tracked and on the parameters of the system. These assumptions, which play a crucial role in the analysis which follows, will be removed in the simulation results where it will be shown how the proposed fault-tolerant controller works properly even in case of state-dependent frequencies.

As a matter of fact, under these assumptions, it is easy to realize that (bearing in mind (12)–(14) and the definition of the slip $s_\omega$)

$$\omega_c(t) + \omega_0(t) = 2\pi f t + (\omega^* + s_\omega^*) t + \omega_0^* := \Omega_1 t + \omega_0^*$$ (16)

as far as faults concerning stator asymmetries are concerned, and

$$\omega_c(t) + \omega_0(t) = 2(\pi f \pm 2k s_\omega^*) t + (\omega^* + s_\omega^*) t + \omega_0^*$$

$$:= \Omega_2, k t + \omega_0^*$$ (17)

as far as faults concerning rotor asymmetries are concerned. In the previous expressions $s_\omega^*$ denotes the (constant) steady
state reached by $s_m$ which turns out to be
\begin{equation}
\dot{s}_m^* := \frac{g_{LM} \hat{r}_L}{\mu_0 f_w^*}
\end{equation}
while $\hat{r}_m^*$ denotes the (unknown) position of the reference frame once the fault occurs.

As final hypothesis about the effect of rotor asymmetries, we assume that there exists a (possibly large) finite integer $N > 0$ such that all the components with frequencies $\Omega_2 \pm k$, $k > N$, are negligible with respect to the first components.

These assumptions allow us to express the deviation of the stator currents values in presence of faults with respect to the un-faulty values as
\begin{align*}
\dot{i}_d & = i_d^{af} + A \sin(\Omega_2 t + \phi) + \sum_{k=1}^{N} [A_k \sin(\Omega_2 k t + \phi_k) \\
& \quad + A_{-k} \sin(\Omega_2 -k t + \phi_{-k})], \\
\dot{i}_q & = i_q^{af} + A \cos(\Omega_2 t + \phi) + \sum_{k=1}^{N} [A_k \cos(\Omega_2 k t + \phi_k) \\
& \quad + A_{-k} \cos(\Omega_2 -k t + \phi_{-k})]
\end{align*}
(19)
namely as the superposition of $(2N + 1)$ harmonic components whose frequencies depend on the specific fault (the first term being due to stator asymmetries, while the last $2N$ terms being due to rotor asymmetries) and with amplitudes and phases depending on the fault severity.

Note that in a realistic scenario frequencies, amplitudes and phases of harmonic components are all uncertain quantities. As a matter of fact, by definition of $\Omega_2$ in (16) and of $\Omega_2 \pm k$ in (17) it is clear that the frequencies depend upon the IM parameters and in particular on the rotor parameter $\alpha$ whose value, as also specified in the last remark of Section 2.3, is in general highly uncertain. Moreover, phases and amplitudes of the fault harmonics are dependent on the fault severity and on the reference frame position when the fault occurs. Clearly, all this quantities cannot be assumed to be a priori known.

We proceed now with some manipulations, with the purpose of expressing in a more convenient way the effect of the fault on the normal (i.e., in the absence of faults) IM dynamics after the occurrence of a fault. As a matter of fact, taking derivatives of (19) it is readily seen that the model of the IM in presence of faults is given by (1) with an exogenous input $V$ equal to
\begin{equation}
V = \left( \begin{array}{c} \Gamma_d \\ \Gamma_q \end{array} \right) w = \Gamma w, \quad \Gamma := \left( \begin{array}{ccc} -\gamma & \ell_1 & \cdots & \ell_N \\ -\ell_1 & -\gamma & \cdots & \ell_N \end{array} \right)
\end{equation}
with
\begin{equation}
\Gamma_k := (-\gamma \ell_{2,k} - \gamma \ell_{3,k}), \quad \ell_k := \omega^* + s^* + \Omega_1, \quad \ell_{2,k} := \omega^* + s_a + \Omega_2, \quad \ell_{3,k} := \omega^* + s_a + \Omega_2 - k.
\end{equation}

Note that, with the above formalism, both stator or rotor or simultaneous faults are allowed, with the first two components of the exosystem state which take into account for stator faults, while the last $4N$ for rotor faults.

To conclude this section it is worth to anticipate that in the next part of the paper we will assume that the initial state of exosystem (20), which as stressed above depends on the specific fault and on its severity, is unknown but ranges within a known, but otherwise arbitrarily large, compact set, denoted $\mathcal{W}$. As the vector of frequencies $\sigma$ is concerned, we will study first the case in which this is perfectly known (which corresponds to require perfect knowledge of the IM parameters) and then the case in which this is uncertain. In the latter case, we will assume the knowledge of a compact set, denoted $\mathcal{F}$, to which the vector $\sigma$ is supposed to belong.

3.2. Reconfiguration strategy

As stressed in the Introduction, the idea behind implicit FTC is that of designing a control unit able to automatically offset the effect of the faults, without need of an explicit FDI process and consequent explicit reconfguration. This objective will be pursued for the IM by means of the control scheme sketched in Fig. 1.

In this scheme, the IFO controller is enriched with a fault compensation unit which, reading the current tracking error $(\dot{i}_d, \dot{i}_q)$, generates additional control inputs $(u_d^f; u_q^f)$ able to offset the effect of any fault belonging to the classes described above. The operation of the fault-tolerant unit is not supervised by any higher level unit as it automatically corrects the control action to achieve fault tolerance. On the contrary, the role of the supervisory unit in Fig. 1 is that of
performing FDI by reading the state of the fault compensation unit. In this perspective, the FDI phase is postponed to that of FTC as it is carried out by looking at the unit which is possibly already compensating the fault.

It is important to stress that the desired goal is to realize the fault compensation unit as a sort of plug-and-play device, whose design is as much independent as possible by the previously designed controller, whose purpose is to achieve the prescribed control objectives in the un-faulty operation mode. As a matter of fact, the only feature required to the control input and with a sufficiently small linear gain function.

For sake of clarity we address separately the design of the fault tolerance control unit in the two cases in which the frequency of the disturbance is known (namely when the matrix $S$ in (20) is known) and that in which it is not.

3.3. Embedding an internal model of the fault

We start from a result of Nikiforov (1998), which permits a suitable parametrization of the pair $(S, \Gamma)$ introduced in Section 3.1. As a matter of fact, in Nikiforov (1998) it is shown that if $S$ is an $n \times n$ matrix having all eigenvalues on the imaginary axis, if $F$ is an $n \times n$ Hurwitz matrix, if $\Gamma$ is a $1 \times n$ vector such that $S, \Gamma$ is observable, and $G$ is an $n \times 1$ vector such that $F, G$ is controllable, the unique solution $T$ of the Sylvester equation

$$TS - FT = \Gamma \Gamma$$  \hspace{1cm} (21)

is nonsingular. From this, it is immediate to realize that

$$T S T^{-1} = (F + G \Phi)$$\ \hspace{1cm} (22)

namely the pair $(S, \Gamma)$ is similar, via the change of coordinates induced by $T$, to the pair $(F + G \Phi, \Phi)$. Using this result twice, it is seen that the two components $V_d, V_q$ of the exogenous input $V$ can be thought of as generated by a pair of systems of the form

$$\dot{\tilde{w}}_d = (F_d + G_d \Phi) \tilde{w}_d, \quad V_d = \Phi_d \tilde{w}_d,$$

$$\dot{\tilde{w}}_q = (F_q + G_q \Phi) \tilde{w}_q, \quad V_q = \Phi_q \tilde{w}_q,$$

in which $\tilde{w}_d := T_d w$ and $\tilde{w}_q := T_q w$. Indeed, we can rewrite these two as one single system

$$\dot{\tilde{w}} = (F + G \Phi) \tilde{w}, \quad V = \Phi \tilde{w}$$ \hspace{1cm} (23)

in which $\tilde{w} = (\tilde{w}_d^T \tilde{w}_q^T)^T$, $T = (T_d^T \ T_q^T)^T$ and $F = \text{blockd}(F_d, F_q), G = \text{blockd}(G_d, G_q), \Phi = \text{blockd}(\Phi_d, \Phi_q)$.

Let $\text{sat}_i(\cdot)$ denote the (piece-wise differentiable) saturation function

$$\text{sat}_i(s) := \text{sgn}(s) \min\{|s|, \lambda|\}$$

and let $\text{dzn}_i(\cdot)$ denote the (piece-wise differentiable) dead-zone function

$$\text{dzn}_i(s) := \text{sat}_i(s) - s.$$  

For a 2-vector-valued argument $v$ we set, with a mild abuse of notation,

$$(\text{sat}_1(v_1), \text{sat}_1(v_2))^T, \quad (\text{dzn}_1(v_1), \text{dzn}_1(v_2))^T.$$  

With this notation in mind, set

$$\tilde{i} := (i_q \ \ i_d)^T \quad u^c := (u^c_\Phi \ u^c_\Phi)^T$$

and note that the dynamics of $\tilde{i}$ can be written in the compact form as

$$\dot{\tilde{i}} = a(x_t, x_s, t) + u^c + \Phi T w,$$

where $a(x_t, x_s, t)$ is a 2-vector-valued function which can readily be obtained from (5) and (8).

Choose now, as fault compensation unit, the following controller (with saturated output):

$$\dot{\tilde{z}} = (F + G \Phi)(\tilde{z} - G \tilde{i}) + K \tilde{i} + G \text{dzn}_i(\Phi \tilde{z} - \Phi G \tilde{i}),$$

$$u^c = \text{sat}_i(\Phi \tilde{z} - \Phi G \tilde{i}),$$  \hspace{1cm} (24)

where $\lambda$ is a positive design parameter and $K := \text{diag}(K_q, K_d)$. It turns out that the fault compensation unit (24) is able to automatically offset the effect of the fault $V = \Phi T w$ for all the allowed initial conditions $w(0)$, if the gain coefficient $k$ which characterizes the indirect field oriented controller (see Proposition 1) and the amplitude $\lambda$ of the saturation function are sufficiently large. In particular, the amplitude of the saturation function, which corresponds to the maximum amplitude of the output of the fault compensation unit, must be compatible with the maximum amplitude of the fault effect, which is given by

$$V_M := \max_{w(0) \in \mathcal{W}} \| \Gamma w \|_{\infty},$$  \hspace{1cm} (25)

where $\mathcal{W}$ represents the compact set where the initial condition $w(0)$ is supposed to lie. Bearing in mind the previous discussion we are able to prove the following proposition.
Proposition 2. Consider system (5), (8) with dynamic feedback control law given by (4), (7), (24), with $K_d = kK_d$, $K_q = kK_q$, $K_\zeta = kK_\zeta$, where $\tilde{K}_d$, $\tilde{K}_q$ and $\tilde{K}_\zeta$ are fixed positive numbers. Let $\lambda$, the output saturation level of (24), be any number such that $\lambda \geq V_M$. Then there exists a number $k^* > 0$ such that, for all $k \geq k^*$, $\|(x(t),x(t))\|$ and $\|\tilde{z}(t) + Tw(t)\|$ asymptotically (and locally exponentially) converge to zero for all $x_1(0) \in \mathbb{R}^3$, $x_4(0) \in \mathbb{R}^4$, $\zeta(0) \in \mathbb{R}^a$ and all $w(t) \in \mathcal{W}$.

Proof. Consider the change of variables
$$
\zeta := \tilde{z} + Tw - G\hat{\chi}.
$$
In the new coordinates (it suffices to consider here only the dynamics of $\tilde{z}$ instead of the whole dynamics of $(x_1,x_4)$)
$$
i = a(x_1,x_4,t) + \text{sat}(\Phi_\chi - \Phi Tw) + \Phi Tw,
$$
$$
\dot{\chi} = F_\chi - G\rho(x_1,x_4,t),
$$
where $\rho(x_1,x_4,t) = a(x_1,x_4,t) + K\tilde{z}$. Now fix $M \geq 2\lambda$, let $k^* = 1$, and $\gamma$ be the lower bound for the gain $k$ and, respectively, the gain coefficient determined in Proposition 1 and note that if $k > k^*$, since $\|v_d\| \leq 2\lambda \leq M$, the restriction (equal to $M$) of the flux/speed subsystem is fulfilled for all $t \geq 0$. As a consequence, since also $\|v_d\| \leq M$, we deduce that $x_1(t),x_4(t)$ exist for all $t$ and
$$
\|x_1(t),x_4(t)\| \leq \frac{\gamma}{k} M.
$$
(28)
Observe now (compare with (5) and (8)) that there exist constants $\ell_1, \ell_2$, independent of $k$, such that
$$
\|\rho(x_1,x_4,t)\| \leq \ell_1 \|x_1(t),x_4(t)\| + \ell_2 \|x_1(t),x_4(t)\|^2
$$
(29)
for all $t \geq 0$. In fact, the term $K\tilde{z}$ in $\rho(x_1,x_4,t)$ cancels out the terms in $a(x_1,x_4,t)$ which depend of $k$. This, the fact that the matrix $F$ is Hurwitz and (28) imply (assuming without loss of generality that $\gamma M/k \leq 1$) that $\chi(t)$ exists for all $t$ and
$$
\|\chi\| \leq q \|\rho(x_1,x_4,t)\|
$$
$$
\leq q(\ell_1 + \ell_2) \|x_1(t),x_4(t)\| \|x_1(t),x_4(t)\| \|x_1(t),x_4(t)\|.
$$
(30)
for some positive $q$. Consider now the function $\text{sat}(\Phi_\chi - \Phi Tw) + \Phi Tw$. Since $\lambda \geq \|\Phi Tw\|$, it turns out that there exists a continuous positive and bounded $\dot{\phi}(t)$ such that
$$
\|\text{sat}(\Phi_\chi - \Phi Tw) + \Phi Tw\| \leq \dot{\phi}(t) \|\text{sat}(\Phi_\chi)\| \leq L\|\chi\|,
$$
(31)
where $L$ is a positive constant. Hence, by the small gain Theorem 1 in Teel (1996) we conclude that if
$$
k > \gamma q(\ell_1 + \ell_2)L
$$
the overall system is globally asymptotically stable. In particular local exponential stability follows from the fact the linear approximation is Hurwitz. Asymptotic convergence of $\|\tilde{z}(t) + Tw(t)\|$ to zero trivially follows from the fact that $\chi(t)$ and $\tilde{i}$ converge to zero.

The statement of the previous result highlights two key features of the fault tolerant unit (24): the first is that, for all possible faults belonging to the classes specified in Section 3.1 (regardless the fault severity) the state $(x_1,x_4)$ of the flux/speed subsystem converges to zero, namely the control objectives are achieved. This means that the control is fault tolerant. The second result is that the state of the fault compensation unit $\zeta$ converges to $-Tw(t)$, namely the state of the exogenous signal is asymptotically reconstructed. This means, as specified in Section 3.1, that the specific fault and its severity can be precisely isolated and evaluated.

We conclude this section with some remarks which shed further light to the result of the previous proposition.

Remark. It is worth noting that the result of Proposition 2 remains true also in case the control action $\hat{u}$ is not provided by the IFO controller specified in Section 2.3 but is generated by a different controller (to this regard see also the discussion at the end of Section 2.3). As a matter of fact it is easy to realize, with an eye to the proof of the previous proposition, that the key feature required for the controller which generates the input $\hat{u}$, is the property of rendering the corresponding closed-loop system input-to-state stable with respect to the matched exogenous input $u^E + V$, with sufficiently large restrictions (compatible with the fault effect to compensate) and with sufficiently small linear gain (to enforce the small gain condition on which the stability proof relies). Any controller yielding such property is suitable for implementing the structure sketched in Fig. 1.

Remark. It is interesting to stress the key role of the saturation function introduced in the output of the fault compensation unit. On one hand, its presence allows us to fulfill the restriction on the input $u^E + V$ which characterizes the system under the IFO controller and hence to use the small gain Theorem of Teel (1996). On the other hand, the saturation plays a role in decoupling the system consisting of the IM and the IFO controller from the dynamics of the fault compensation unit. In this regard, note that a key point in the proof of the previous proposition is the fact that the state $(x_1,x_4)$ can be rendered asymptotically small (see (28)), which in turn implies that the quadratic function $a(x_1,x_4,t)$ in (29) can be asymptotically dominated by a linear function (see (30)). This fact, which is crucial to enforce the small gain condition, holds precisely because of the presence of a saturation function, which renders the bound (28) fulfilled independently of the $\chi$-dynamics.

3.4. Adaptive frequency estimation

In this section we address the case in which the vector $\sigma$ of frequencies which characterize the fault in the current dynamics are not perfectly known, but are supposed to range on a fixed (though arbitrarily large) compact set $\mathcal{F}$. The theory presented in this section takes a major source of inspiration from the general theory about regulation with adaptive
internal model of Serrani, Isidori, and Marconi (2001), but proposes however some nontrivial modifications which can be seen as an interesting development of the work (Serrani et al., 2001).

Before entering into the details of the analysis, it is worth noting that the lack of knowledge of the frequencies to be compensated is a problem in real implementation. As a matter of fact, looking at the definitions of $\Omega_1$ in (16) and $\Omega_{2,3,4}$ in (17), it is immediately acknowledged that the frequencies of the fault depend on the steady state value of the slip variable $s$, which, in turn, depends on the rotor parameter $\alpha$ and the latter, as also stressed before, is in general highly uncertain.

Since the control law (24) cannot be implemented, because $\Phi$ depends on the uncertain vector $\sigma$, we use instead a control law of the form
\[
\dot{\zeta} = (F + G\hat{\Phi})(\zeta - \hat{G}\hat{\zeta}) - G\hat{K}\hat{G}\zeta + G\, d\, z\, (\hat{\Phi}\zeta - \hat{\Phi}\hat{G}\hat{G}\zeta),
\]
\[
\theta = \text{sat}_e(\hat{\Phi}\zeta - \hat{\Phi}\hat{G}\hat{G}\zeta),
\]
where $\zeta = (\zeta_{yz}^T \zeta_{\psi}^T)^T$, $\hat{\Phi} = \text{blockd}(\hat{\Phi}_d, \hat{\Phi}_q)$, is an estimate updated according to the following dynamics:
\[
\dot{\hat{\Phi}}_d = d\, z\, (\hat{\Phi}_d) - \rho \tilde{\hat{G}}^T \hat{\Phi}_d,
\]
\[
\dot{\hat{\Phi}}_q = d\, z\, (\hat{\Phi}_q) - \rho \Psi^* \hat{\Phi}_q,
\]
in which $\ell$ and $\rho$ are positive design parameters. As in the previous section, dealing with the case of known frequencies, it is possible to prove that the indirect field oriented controller (4), (7) with the adaptive fault compensation unit (32), (33) is able, for sufficiently large value of $\lambda$, $\ell$ and $k$, to achieve the control objectives while offsetting the effect $V$ of any fault.

The next proposition provides the desired result.

**Proposition 3.** Consider system (5), (8) with dynamic feedback control law given by (4), (7) (32) and (33), with $K_d = k\hat{G}\hat{G}$, $K_q = k\hat{G}\hat{G}$, $K_{\psi} = k\hat{G}\hat{G}$ where $\hat{G}\hat{G}$ and $K_{\psi}$ are fixed positive numbers. Suppose that the vector $\sigma$ and the initial state $w(0)$ range within fixed compact sets $\mathcal{F}$ and $\mathcal{W}$, respectively. Let $\ell$ and $\rho$ be arbitrary constants with $\ell$ (the amplitude of the dead-zone functions which characterize the adaptation law (33)) such that $\ell \geq \Phi_M := \max_{\sigma \in \mathcal{F}}(\|\Phi_d\|, \|\Phi_q\|).

Then there exist $\lambda > 0$ and $k^* > 0$ such that, for all $k \geq k^*$, $\|(x_k(t), x_q(t))\|$ and $\|\zeta(t) + Tw(t)\|$ asymptotically (and locally exponentially) converges to zero for all $x_k(0) \in \mathbb{R}^3$, $x_q(0) \in \mathbb{R}^4$, $\zeta(0) \in \mathbb{R}^n$, $\Phi(0) \in \mathbb{R}^n$ and all $w(0) \in \mathcal{W}$, $\sigma \in \mathcal{F}$.

**Proof.** For convenience the proof is divided in two parts. In the first part, it is shown that sufficiently large values of $\lambda$ and $k$ guarantee that the trajectories are bounded and the saturation function enters the linear region in finite time. Then, in the second part, a Lyapunov argument is used to show that fault tolerance is achieved, namely the state $(x_k, x_q)$ asymptotically decays to zero.

Consider again the change of variable (26). The inputs $v = (v_d, v_q)$ to (5)–(8), in the new coordinates, read as
\[
v = \text{sat}_s(\hat{\Phi}_d - \hat{\Phi}T w) + \Phi Tw.
\]
Since $\|\Phi Tw\| \leq V_M$ (with $V_M$ defined in (25)) and $|\text{sat}_s(s)| \leq \lambda$ for all $s$, assuming without loss of generality that $\lambda \geq V_M \geq 1$, it turns out that $\|v\| \leq 4\lambda$. This implies that (see Proposition 1), if $k$ is large enough, $x_k(t), x_q(t)$ exist for all $t$ and
\[
\|(x_k, x_q(t))\| \leq \frac{4\lambda}{k} \quad \text{taking } k = k'\lambda.
\]
Even though the control law has changed, from (24) to (32), the dynamics of $\hat{\chi}$ still has the form given by the second equation of (27). Bearing in mind the fact that $F$ is Hurwitz, the bound (29), and assuming without loss of generality $k > 1$, the estimate (35) also implies that $\chi(t)$ exists for all $t$ and there exists a $\delta > 0$, not dependent on $\lambda$, such that $||\chi|| \leq \frac{\delta}{k'}$.

Hence, by definition of $\chi$,
\[
||\hat{\chi}|| \leq \|\chi + Tw + G\hat{G}\hat{\chi}\|
\]
\[
\leq \|Tw\| + \|\chi + G\hat{G}\hat{\chi}\|
\]
\[
\leq \|Tw\| + \frac{\delta + 4\|G\|\gamma}{k'} \leq m.
\]

In the previous expression $m$ is a positive constant not dependent on $k'$ (assuming $k' \geq 1$) and not dependent on $\lambda$.

We now focus on the $\Phi$ dynamics (33). Since $|\text{sat}_s(s)| \leq \ell$ for all $s \in \mathbb{R}$ and
\[
||\hat{\chi}^T|| \leq \|(x_k, x_q(t))\||\hat{\zeta}|| \leq \frac{4m}{k'}
\]
it is easy to realize that $\hat{\Phi}$ is bounded and the following asymptotic estimate holds:
\[
||\hat{\Phi}|| \leq \ell + \frac{4\rho\gamma m}{k'} \leq 2\lambda,
\]
where the last inequality holds provided $k' \geq 4\rho\gamma m$. This means that the argument of the saturation function in (34) satisfies an asymptotic estimate of the form
\[
||\hat{\Phi}_d - \hat{\Phi}T w|| < 2\lambda \left( \frac{\delta}{k'} + \|Tw\| \right) \leq n
\]
in which $n$ is a positive number not dependent on $\lambda$ and $k'$ (as $k' \geq 1$). In view of this, we choose $\lambda \geq n$. This implies that there exists a $T' > 0$ such that for all $t \geq T'$
\[
\text{sat}_s(\hat{\Phi}(t) - \hat{\Phi}T w(t)) + \Phi Tw(t)
\]
\[
= \hat{\Phi}(t) - \hat{\Phi}(t)Try(t) + \Phi Tw(t)
\]
\[
= \hat{\Phi}(t)\zeta(t) + \hat{\Phi}(t) - \hat{\Phi}(t)G\hat{G}(t),
\]
where $\hat{\Phi} := \hat{\Phi} - \Phi$.

(37)
This completes the first part of the proof. Note that the previous discussion, in addition to proving that the saturation function enters in finite time the linear region, has also shown that the states of the flux and speed subsystems can be rendered arbitrarily small by increasing \( k \) (see (35)) and that the estimate \( \hat{\phi} \) is bounded by a positive number (see (36)).

We proceed now to prove that \( x_{1}(t) \) and \( x_{2}(t) \) asymptotically decay to zero. The overall system consists of the flux subsystem (5), of the speed subsystem (8), with

\[
v = \dot{\phi}z + \Phi \chi - \Phi G \tilde{t}
\]

of the fault compensation unit, whose dynamics in the \( \chi \)-coordinates is described by the second equation in (27), and of the adaptation law (33) which in the new error coordinates (37) reads as

\[
\dot{\hat{\phi}}_{d} = d_{zn}(\hat{\phi}_{d} + \phi_{d}) - \tilde{\phi}_{d} \xi_{d}^{T} \xi_{d},
\]

\[
\dot{\hat{\phi}}_{q} = d_{zn}(\hat{\phi}_{q} + \phi_{q}) - n_{1}\xi_{q}^{T} \xi_{q}.
\]

(38)

We construct the Lyapunov function for the whole system at different stages. First, consider the flux subsystem and the Lyapunov function \( V_{f} = \frac{1}{2} x_{1}^{T} x_{1} \). Taking derivatives along (5), simple computations show that for a large value of \( k \)

\[
\dot{V}_{f} \leq - n_{1} \|x_{f}\|^{2} - (k \tilde{K}_{f} - n_{2}) \tilde{f}_{d}^{T} \tilde{f}_{d} + \tilde{\phi}_{d} \tilde{x}_{d}^{T} \tilde{f}_{d} + n_{3} \|\tilde{f}_{d}\| \|\chi\|
\]

for some positive numbers \( n_{1}, n_{2}, n_{3} \). Consider now the speed subsystem (8) and set \( x_{c} = (x_{1}^{T}, x_{2}^{T}) \), where \( x_{1}^{T} := (\tilde{\omega}_{1}, \omega) \) and \( x_{2}^{T} := (\xi, \eta) \). For this system consider the Lyapunov function

\[
V_{s} = x_{2}^{T} P_{1} x_{2} + \frac{1}{2} x_{1}^{T} x_{1}^{*} x_{2}^{2} \quad \text{with} \quad P_{1} \text{ defined in (A.4). Bearing in mind estimate (35) and noting that there exist positive numbers } a, b_{0}, b_{1} \text{ such that}
\]

\[
\|A_{d}(x_{f}, t)\| \leq a\|x_{f}\|, \quad \|B_{s}(x_{f}, t)\| \leq b_{0} + b_{1}\|x_{f}\|
\]

it is easy to obtain that, for a sufficiently large value of \( k \), there exists a \( T_{1}^{*} \) such that for all \( t \geq T_{1}^{*} \)

\[
\dot{V}_{s} \leq - q_{1}\|x_{c}\|^{2} - (k \tilde{K}_{q} - q_{2}) \eta^{2} + q_{3}\|x_{f}\| \|x_{c}\| + \eta \tilde{\phi}_{q} \tilde{q}_{q}
\]

\[
\quad + q_{4}\|\eta\| \|\xi\|
\]

for some positive numbers \( q_{i}, i = 1, \ldots, 4 \). Consider now the \( \chi \)-dynamics and define \( V_{\chi} = \chi^{T} P_{\chi} \chi \),

\[P_{1} F + F^{T} P_{1} = -I.\]

Recalling (29) and (35), it is easy to conclude that for a sufficiently large \( k \), there exists a \( T_{2}^{*} \) such that for all \( t \geq T_{2}^{*} \)

\[
\dot{V}_{\chi}(t) \leq - \|\chi(t)\|^{2} + \epsilon_{1}\|\chi(t)\| \|(x_{f}(t), x_{c}(t))\|.
\]

for some positive \( \epsilon_{1} \). Finally define

\[
V_{\phi} := \tilde{\phi}_{d} \tilde{f}_{d}^{T} / 2 \rho + \tilde{\phi}_{q} \tilde{q}_{q}^{T} / 2 \rho
\]

so that, differentiating along (38),

\[
\dot{V}_{\phi} = \tilde{\phi}_{d} d_{zn}(\hat{\phi}_{d} + \phi_{d}) / \rho + \tilde{\phi}_{q} d_{zn}(\hat{\phi}_{q} + \phi_{q}) / \rho
\]

\[
- \tilde{\phi}_{d} \tilde{x}_{d}^{T} \tilde{f}_{d} - \tilde{\phi}_{q} \tilde{q}_{q}^{T}.
\]

For all \( \ell \geq |a| \), the graph of the function \( d_{zn}(s + a) \) is (second quadrant)/(fourth quadrant), and therefore

\[
s d_{zn}(s + a) \leq 0 \quad \text{for all } s \in \mathbb{R}.
\]

Hence, since by hypothesis \( \ell \geq \Phi_{M} \),

\[
\dot{V}_{\phi} \leq - \tilde{\phi}_{d} \tilde{x}_{d}^{T} \tilde{f}_{d} - \tilde{\phi}_{q} \tilde{q}_{q}^{T}.
\]

Define now the Lyapunov function for the whole system as

\[
\dot{W}(x_{f}, x_{c}, \chi, \hat{\phi}) := V_{f}(x_{f}) + \epsilon_{1} V_{s}(x_{s}) + \epsilon_{2} V_{\chi}(\chi) + V_{\phi}(\hat{\phi})
\]

with

\[
\epsilon_{1} \leq \frac{q_{1} n_{1}}{q_{2}^{2}}, \quad \epsilon_{2} \leq \frac{n_{1} + q_{1}}{\ell_{1}}.
\]

A simple application of the Young’s inequality shows that for sufficiently large \( k \)

\[
\dot{W}(t) \leq - r_{1} \|(x_{f}(t), x_{c}(t))\| - r_{2} \|\chi(t)\|
\]

\[
\forall t \geq \max \{T_{1}^{*}, T_{2}^{*}\}
\]

for some positive \( r_{1} \) and \( r_{2} \). This, by the LaSalle Theorem, implies that the trajectories of the closed-loop system converge toward the largest invariant set characterized by \( x_{f} = 0 \), \( x_{s} = 0 \) and \( \chi = 0 \). This concludes the second part of the proof. \( \square \)

Few remarks to comment the previous proposition are in order.

**Remark.** The adaptation law chosen in (33) differs from that proposed in Serrani et al. (2001) for the presence of the dead-zone functions \( d_{zn}(\cdot) \) which is motivated by the fact that the output of the fault compensation unit is saturated. As a matter of fact it is interesting to note, with an eye to the proof of the previous proposition, that the role of \( d_{zn}(\cdot) \) consists of keeping the estimate \( \hat{\phi} \) bounded with a bound which is not dependent on \( k \) (see (36) and the analysis just before). This indeed is crucial to show that in finite time the saturation function which characterizes the output of the fault compensation unit definitely enters the linear region.

**Remark.** Note that the proposition is not conclusive about the asymptotic convergence of matrix \( \hat{\phi} \) to \( \phi \). In this respect it can be easily proved (following an analysis similar to that in Serrani et al. (2001) that \( \hat{\phi} \) converges to a fixed matrix which is, in general, different from \( \phi \) unlike the case in which all the frequencies \( \sigma \) are excited by the fault (namely if the initial condition \( w(0) \) is such that the signals \( T w(t) \) contains all the frequencies in \( \sigma \)). This in general is not true as it happens only in case of simultaneously stator and rotor asymmetries.

### 3.5. Fault detection and isolation

According to the controller structure sketched in Fig. 1, the FDI can be performed by checking the state of the fault compensation unit which automatically offsets the fault effect. The detection phase, whose purpose is to identify the occurrence of *some* fault, can be easily carried out by comparing \( \|\tilde{\chi}(t)\| \) with a suitably tuned threshold. As a matter of fact by Proposition 2 (or equivalently Proposition 3...
in the adaptive scenario) \( \zeta(t) \) asymptotically converge on \(-T_{W}\) which is zero in the un-faulty case and different from zero when a fault occurs.

Also the isolation procedure, which consists of the identification of the occurrence of a specific fault, is possible in this setup, as indicated in the following example. In particular consider the problem of distinguishing two kinds of fault regarding stator or rotor asymmetries. Again according to the model presented in Section 3.1, it is clear that faults due to stator asymmetries are represented by the first two components of the exogenous state \( w(t) \) while faults due to rotor asymmetries are represented by the last \( N - 2 \) components of \( w(t) \). In view of this, setting

\[
\hat{w}(t) := T^{-1} \zeta(t)
\]

it is possible to define residual signals as

\[
r_{s}(t) = \begin{cases} 
1 & \text{if } \|\hat{w}(t)\|_{1,2} \geq T_{s}, \\
0 & \text{otherwise,}
\end{cases}
\]

\[
r_{r}(t) = \begin{cases} 
1 & \text{if } \|\hat{w}(t)\|_{3,5} \geq T_{r}, \\
0 & \text{otherwise,}
\end{cases}
\]

where \( T_{s} \) and \( T_{r} \) are two positive thresholds. The two residual signals \( r_{s} \) and \( r_{r} \) correspond to faults due to stator asymmetries and, respectively, faults due to rotor asymmetries. We will not take up in this paper important issues, such as how to robustly choose the thresholds in order to avoid the occurrence of false alarms or missed diagnosis, which indeed will be dealt with in future works.

It is important to note that the previous isolation strategy cannot be implemented as such in case the vector \( \sigma \) of exogenous frequencies is not perfectly known. As a matter of fact in such a case the matrix \( T \) solution of (21), which depends on \( \sigma \), is not known and the exogenous variable estimate \( \hat{w} \) cannot be computed. In this scenario a more sophisticated isolation algorithm should be identified by using signal processing algorithms. This issue will not be developed further in this paper and will be the subject of future research.

### 4. Experimental and simulation results

The effectiveness of the adaptive implicit FTC algorithm described in Sections 3.3 and 3.4 in presence of mechanical faults has been tested in the experimental activity carried out at the Laboratory of Automation and Robotics (LAR) of University of Bologna. Specifically the experimental setup consists of an IM, on which the mechanical fault is implemented as described later, connected to a brush-less motor, acting as a load torque generator, by means of an adaptive joint able to compensate for angular, axial and radial offsets. The IM is a commercial asynchronous three phases 1.1 kW motor with 50 Hz and 380–410 V power supply whose electrical and mechanical parameters are shown in Table 1. The experimental setup is then completed with a power inverter with 540 V DC-link voltage and a control board based on a DSP TMS320C32 designed and developed within LAR. The sampling time for controller implementation is set to 400 \( \mu \)s. The DSP performs data acquisition, generates speed and flux references, implements the control algorithm and generates the PWM inverter commands. The control board is connected to a standard PC used for DSP programming and for data acquisition and displaying.

The stator current and the motor angular speed, processed by the adaptive algorithm presented in the paper, are acquired by means of commercial Hall-type sensors yielding a 0–10 V signal which is proportional to the instantaneous value of a AC current signal (0–50 A) and by a two poles commercial resolver (6 V, 10 kHz, with a transformation rate 0.28 ± 10%\) with an encoder simulation output (1024 ppr).

The IM has been damaged by introducing a mechanical rotor fault. Specifically five of the 28 rotor bars have been holed in order to simulate a broken bar rotor failure, see Fig. 2 The diameter of each hole is 4 mm.

In all the experimental results which will be presented in the following, we have fixed a constant speed reference \( \omega = 100 \text{ rad/s} \) and a constant load torque \( T_L = 6 \text{ N m} \) applied by means of the brush-less motor. Furthermore the IFO controller described in Section 2.3 has been tuned following the constructive procedure illustrated in Peresada, Tonielli,  

![Fig. 2. The rotor of the IM with five bars broken by means of two holes each bar.](image)

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator inductance ( L_s )</td>
<td>( L_s )</td>
<td>0.663</td>
<td>H</td>
</tr>
<tr>
<td>Rotor inductance ( L_r )</td>
<td>( L_r )</td>
<td>0.663</td>
<td>H</td>
</tr>
<tr>
<td>Mutual inductance ( L_m )</td>
<td>( L_m )</td>
<td>0.627</td>
<td>H</td>
</tr>
<tr>
<td>Stator resistance ( R_s )</td>
<td>( R_s )</td>
<td>6.25</td>
<td>( \Omega )</td>
</tr>
<tr>
<td>Rotor resistance ( R_r )</td>
<td>( R_r )</td>
<td>6.26</td>
<td>( \Omega )</td>
</tr>
<tr>
<td>Rotor inertia ( J )</td>
<td>( J )</td>
<td>0.0024</td>
<td>Kg m^2</td>
</tr>
<tr>
<td>No. of pole pairs ( n_p )</td>
<td>( n_p )</td>
<td>2</td>
<td>–</td>
</tr>
<tr>
<td>Rated load</td>
<td></td>
<td>7</td>
<td>N m</td>
</tr>
<tr>
<td>Rated speed</td>
<td></td>
<td>1500</td>
<td>rpm</td>
</tr>
</tbody>
</table>

Table 1

Parameters of the IM adopted for the experimental activity
Table 2
IFO controller gains adopted for experimental activities

<table>
<thead>
<tr>
<th>$K_w$</th>
<th>$K_d$</th>
<th>$K_q$</th>
<th>$K_z$</th>
<th>$K_a$</th>
<th>$K_f$</th>
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<tr>
<td>120</td>
<td>300</td>
<td>300</td>
<td>500</td>
<td>90</td>
<td>7200</td>
</tr>
</tbody>
</table>

Fig. 3. Experimental results. Current tracking error $\tilde{i}_d(t)$ in case of rotor failure with the IM controlled via the standard IFO controller (upper plot) and using the implicit FT controller (lower plot).

and Morici (1999) and the control parameters thus obtained are shown in Table 2.

As far as the fault compensation unit described in Sections 3.3 and 3.4 is concerned, we have fixed $\lambda$, the amplitude of the saturation function, and $\zeta$, the amplitude of the deadzone function, respectively to $\lambda = 2000$ and $\zeta = 2500$. These large values have been chosen in order to test the fault tolerance even in case of very severe failures. Furthermore, the tuning of the fault compensation unit (32)–(33) has been completed taking $\rho = 5$ and the controllable pair $(F, G)$ with $F$ an Hurwitz diagonal matrix with eigenvalues in $\{-10, -20, -30, -40, -50, -60\}$ and

$$G = \begin{bmatrix} 1 & 0 & 3 & 0 & 5 & 0 \ 0 & 2 & 0 & 4 & 0 & 6 \end{bmatrix}^T.$$  

Figs. 3, 4 and 5 report, respectively, the steady state current tracking error $\tilde{i}_d(t)$, the steady state current tracking error $\tilde{i}_q(t)$ and the steady state speed tracking error $\tilde{\omega}(t)$ in case just the IFO controller is present in the loop (upper plots) and in case the IFO controller is enriched with the fault compensation unit (lower plots). It is worth to note that the presence of the failure due to the broken bars, if not compensated by means of the fault tolerance unit, generates a quite large steady-state tracking error for the current $i_d$ (in particular $\tilde{i}_d(t)$ shows a mean value equal to 0.1 and oscillations of amplitude 0.4) which in turn yields large deviation of the speed tracking error. In this respect the effectiveness of the fault compensation unit in compensating the effect of the faults and recovering the control objectives is quite evident in these figures.

In order to stress how the information extracted by the state behavior of the fault compensation unit can be useful in order to perform detection of the occurred (and compensated) fault (see Section 3.5), we have reported in Fig. 6 the behavior of the voltages $\sigma_{i_d}^E$ and $\sigma_{i_q}^E$ when the implicit fault compensation algorithm is run in presence of an un-faulty IM (upper plot) and of the IM with broken bars.
Fig. 6. Experimental results. Left plots: the output $u_{fc}d(t)$ of the fault compensation unit with an un-faulty IM (upper plot) and with the faulty IM (lower plot). Right plots: the output $u_{fc}q(t)$ of the fault compensation unit with an un-faulty IM (upper plot) and with the faulty IM (lower plot).

(lower plots). In this respect it is quite evident that the behavior of these additional control inputs yields a robust information about the presence of the fault which can be used for fault detection and isolation. In particular, as stressed in Section 3.5, the comparison of $u_{fc}d$ and $u_{fc}q$ with a fixed (or suitably adapted) threshold can be used in order to detect if the fault compensation unit is working to compensate for oscillations and hence if a fault is occurred.

To conclude this section we present few simulation results aiming to test the performances of the algorithm also in presence of stator faults due to static eccentricities which, at the state of the art, are not yet implemented in the experimental setup above described.

The same electrical and mechanical values presented in Table 1 have been adopted as nominal parameters and the simulation results have been obtained assuming parametric uncertainties up to 20%. Moreover, the same tuning of the IFO controller and of the fault compensation unit used for the experimental activity has been chosen.

According to the theory presented in Section 3.1, to simulate a stator and rotor failure the stator currents have been corrupted as in (19) taking $N = 1$ and assuming $\phi = \phi_1 = \phi_{-1} = 0$ and $\Omega, \Omega_1$ and $\Omega_{-1}$ computed using (16) and (17). The amplitudes $A, A_1$ and $A_2$ have been set to zero or to a positive number depending on the kind of simulated fault (stator or rotor). Finally the stator currents and angular speed, which are processed by the control algorithm, have been corrupted with a Gaussian white noise with zero mean and standard deviation $\pm 0.2$ A and $\pm 0.3$ rad/s, respectively.

In all the experiments presented in the following the IM is simulated in the un-faulty condition before $t = 2$ s and in different faulty scenarios from $t = 2$ s. Moreover the fault compensation unit is always initially not activated and it is inserted in the control loop, with the only exception of the first experiment, at the time $t = 1.2$ s. In Fig. 7, the behavior of speed and flux tracking errors is shown in case, respectively, of rotor fault (with $A_1 = A_{-1} = 0.1$) and of stator fault (with $A = 0.1$) with the fault compensation unit activated in the loop. In these figures one can recognize the transient at $t = 2$ s due to the occurred fault and how
force desired flux and speed profiles, can be “enriched” with an internal model unit in order to achieve fault tolerance and also fault FDI. The design of the internal model unit can be considered independent from that of the stabilizing IFO unit as only the current gains are eventually required to be re-tuned. We have shown how the internal model unit can be designed in order to have “global” tracking of the desired references and “semi-global” tolerance to possible faults. Experimental and simulation results have been presented in order to show the effectiveness of the approach.

Acknowledgements

The authors thank Prof. Sergei Peresada and Prof. Alberto Tonielli for their useful suggestions during the preparation of this work and Ing. Davide Bagnara for the support given in the experimental activity presented in the paper carried out at Laboratory of Automation and Robotics of University of Bologna.

Appendix A. Proof of Proposition 1

We first present two propositions, regarding respectively the flux subsystem (5) and the speed subsystem (8), which are instrumental to prove Proposition 1. The proof of the first proposition is omitted since straightforward.

Proposition A.1. Set $K_d=k\hat{K}_d$, where $\hat{K}_d$ is a positive fixed constant. Then there exists a number $k^*_1>0$, independent of $\omega(t)$ and $s_\omega(t)$, such that for all $k \geq k^*_1$ system (5) is ISS without restriction on the input $v_d$ and on the initial state and linear gain functions. In particular, there exist numbers $\gamma_f>0$ and $\gamma^*_f>0$ such that, for each $x^0_d \in \mathbb{R}^d$ and each measurable $v_d$, the solution of (5) with $x^0_f(0)=x^0_f$ exists for all $t \geq 0$ and satisfies

$$
\|x_f\|_\infty \leq \max \left\{ \gamma_f |x^0_d|; \frac{\gamma_f}{k} \|v_d\|_\infty \right\},
$$

$$
\|x_f\|_a \leq \frac{\gamma_f}{k} \|v_d\|_a.
$$

(A.1)

Proposition A.2. Set $K_q=k\hat{K}_q$ and $K_z=k\hat{K}_z$, where $\hat{K}_q$ and $\hat{K}_z$ are positive fixed constants. Then there exist numbers $k^*_2>0$ and $\Lambda>0$, such that for all $k \geq k^*_2$ system (8) is ISS with restrictions ($\Lambda, \infty$) on the inputs $(x_f, v_q)$, without restriction on the initial state, and linear gain functions. In particular there exist numbers $\gamma_s>0$ and $\gamma^*_s>0$ such that, for each $x^0_q \in \mathbb{R}^q$ and each measurable $(x_f, v_q)$, the solution of (8) with $x_q(0)=x^0_q$ exists for all $t \geq 0$ and satisfies

$$
\|x_q\|_\infty \leq \max \left\{ \gamma^*_s |x^0_q|, \gamma_s \max \left\{ \|x_f\|_\infty, \frac{1}{k} \|v_q\|_\infty \right\} \right\},
$$

$$
\|x_q\|_a \leq \gamma_s \max \left\{ \|x_f\|_a, \frac{1}{k} \|v_q\|_a \right\}.
$$

5. Conclusions

In this paper we have presented a new idea for dealing with fault-tolerant control systems design presenting the design of a fault tolerant control unit for induction motors suitable for dealing with mechanical faults. We have shown how an Indirect Field Oriented controller processing the currents and the angular velocity of the IM in order to en-

the fault tolerance is asymptotically achieved by the fault compensation unit.
Proof. The proof is a consequence of the small gain theorem. In particular it is worth partitioning the state variable \( x_s = (x_1^s, x_2^s) \), with \( x_1^s = (\tilde{T}_L, \tilde{\omega})^T \) and \( x_2^s = (\xi, \eta)^T \), and considering the 4-dimensional system (8) as the feedback interconnection of the 2-dimensional subsystems

\[
\begin{align*}
\dot{x}_1^s &= \begin{bmatrix} A_{11}^s & A_{12}^s \end{bmatrix} x_1^s + \begin{bmatrix} A_{11}^{2s} & A_{12}^{2s} \end{bmatrix} (x_s, t)\quad \text{with the 2-dimensional subsystem} \\
\dot{x}_2^s &= \begin{bmatrix} A_{21}^s & A_{22}^s \end{bmatrix} x_2^s + \begin{bmatrix} A_{21}^{2s} & A_{22}^{2s} \end{bmatrix} (x_s, t)
\end{align*}
\]

where the matrices \( A_{ij}^s \) and \( A_{ij}^{2s} \) are easily obtained from (9) and (10). The first subsystem with state \( x_1^s \) can be shown to be ISS without restriction on the initial state, nonzero restriction on the inputs \( (x_2^s, x_2^s) \) and linear gain function. As a matter of fact, since \( A_{11}^s \) is Hurwitz, consider the candidate ISS-Lyapunov function \( V = x_1^s x_2^s + P_1 x_2^s \) with \( P_1 > 0 \) solution of

\[
P_1 A_{11}^s + A_{11}^{2s} P_1 = -I \tag{A.4}
\]

and note that there exist positive constants \( a \) and \( b_0, b_1 \) such that

\[
\| A_{11}^s (x_s, t) \| \leq a \| x_s \|, \quad \| A_{11}^{2s} (x_s, t) \| \leq a \| x_s \|, \\
\| B_{11}^s (x_s, t) \| \leq b_0 + b_1 \| x_s \|.
\]

Taking derivatives of \( V \) along the solution of (A.2) we get

\[
\dot{V} = -\| x_1^s \|^2 + 2 x_1^s P_1 (A_{11}^s (x_s, t) x_1^s + (A_{11}^{2s} + A_{12}^{2s} (x_s, t)) x_2^s + B_1^s (x_s, t) x_2^s)
\]

\[
\leq -\| x_1^s \|^2 + 2 \| P_1 \| \| x_1^s \| \| x_1^s \| (a \| x_1^s \| \| x_1^s \| + \| A_{11}^{2s} \| \| x_1^s \|)
\]

\[
= -\| x_1^s \| \| x_1^s \| (a + b_0 \| x_1^s \| + b_1 \| x_1^s \|^2).
\]

Now fix \( A' > 0 \) so that \( A' \| P_1 \| < \frac{1}{4} \) and note that \( \| x_s \| < A' \) implies

\[
\dot{V} \leq -\frac{1}{4} \| x_1^s \|^2 + (\ell_1 + \ell_3 A') \| x_1^s \| \| x_1^s \| + (\ell_2 + \ell_3 A') \| x_1^s \| \| x_1^s \|
\]

for some positive numbers \( \ell_i, i = 1, \ldots, 3 \). From this and Lemma 3.3 in [Teel (1996)] it turns out that the system (A.2) is ISS without restriction on the initial state, restrictions \( \infty, A' \) on the inputs \( (x_2^s, x_2^s) \) and linear gain functions. In particular there exists \( \gamma_s > 0 \) such that

\[
\| x_1^s \| \leq \gamma_s \| x_2^s \|, \quad \| x_2^s \| \leq \gamma_s \| x_2^s \|.
\]

In a similar way, it can be shown that system (A.3) is ISS without restriction on the initial state, nonzero restriction on the inputs \( (x_2^s, x_2^s) \) and linear gain function, whose coefficient can be arbitrarily lowered by increasing \( k \). To this end, observe that the real part of the eigenvalues of the matrix \( A_{22}^{2s} \) depends linearly on \( 1/k \). In view of this there exist symmetric positive definite matrices \( P_2 \) and \( Q \) such that

\[
P_2 A_{11}^{2s} + A_{11}^{2s} P_2 = -2k P_2 - Q \tag{A.6}
\]

for some positive \( k \). Consider now the candidate ISS Lyapunov function \( V = x_2^s P_2 x_2^s \) whose derivative along (A.3) satisfies (observe that bounds like (A.5) also hold for \( A_{21}^{2s} \) and \( B_2^s \))

\[
\dot{V} \leq -\frac{1}{k} \| P_2 \| \| x_2^s \|^2 - x_2^s Q x_2^s + \| x_2^s \| (q_0 \| x_2^s \| \| x_2^s \|
\]

\[
+ q_1 \| x_2^s \| \| x_2^s \| + q_2 \| x_2^s \| + q_3 \| x_2^s \| + q_4 \| x_2^s \|
\]

for some positive numbers \( q_i, i = 0, \ldots, 5 \), and fix \( A'' > 0 \) such that \( q_0 A'' < \| Q \| \). Hence \( \| x_2^s \| < A'' \) implies that

\[
\dot{V} \leq -\frac{1}{k} \| P_2 \| \| x_2^s \|^2 + \| x_2^s \| (q_1 + q_2 A'') \| x_2^s \|
\]

\[
+ (q_3 + q_4 A'') \| x_2^s \| + q_4 \| x_2^s \|
\]

from which it is easy to conclude, again by Lemma 3.3 in [Teel (1996)], that system (A.2) is ISS without restriction on the initial state, restrictions \( \infty, A'', \infty \) on the inputs \( (x_2^s, x_2^s, v_q) \) and linear gain functions. In particular there exists \( \gamma_s'' > 0 \) such that

\[
\| x_1^s \| \leq \gamma_s'' \max \{\| x_2^s \|, \| x_2^s \|, \| v_q \|\}.
\]

In view of this, a simple application of the small gain Theorem 1 in [Teel (1996)] proves the claim of the proposition with \( k_1^* \geq \gamma_s'' \max \{1, 1/\gamma_s'' / k_1^* \}, A \leq \min \{A', A''\} \) and \( \gamma_s \geq 2 \max \{\gamma_s / k_1^*, \gamma_s'' / k_1^* \}. \]

Bearing these results in mind we prove now Proposition 1. The idea is to act on \( k \) in order to force \( x_2 \) to fulfill the restriction \( A \) on an interval \( [T^*, \infty) \) and to see the overall system as cascade connection of two ISS systems. For this we need first to make sure that the solution exists on any interval of the form \( [0, T^*] \), i.e. that the system does not have finite escape time. To this end note that, if solutions (hence, in particular, \( o(t) \) and \( s(t) \)) are defined on a interval \( [0, T^*] \), \( \| x_2(t) \| \) is bounded by the fixed quantity \( \max \{\| x_2(t) \|, \ell_1 / k \} \). On the other hand, the growth of the right-hand side of (8) is affine in \( x_2 \), with coefficients only depending on bounds on \( |x_2(t)| \) and \( |v_q(t)| \). Thus, on the interval \( [0, T^*] \), \( |x_2(t)| \) is guaranteed to be bounded by an exponentially growing function which only depends on \( \| x_2(t) \|, \| x_2(t) \|, \| v_q(t) \|, \| v_q(t) \| \). As a consequence, solutions exist on any interval of the form \( [0, T^*] \), i.e. no finite escape time can occur.

Now note that, by Proposition A.1, \( \| v_q \| < M \) and \( k \geq k_1^* \) imply that

\[
\| x_2 \| \leq \gamma_1 / k_M.
\]
Hence, fixing \( k^* > \gamma_1 M/\Delta \), it turns out that there exists \( T^* > 0 \) such that \( ||x_i(t)|| < \Delta \) for all \( t \geq T^* \), namely the input \( x_i \) of the speed subsystem fulfills the restriction \( \Delta \) in finite time. From this the claim of the proposition follows by standard cascade arguments as a consequence of Propositions A.1 and A.2. In particular the fact that the asymptotic gain is proportional to \( 1/k \) follows from gain composition.

References


Claudio Bonivento was born in Bologna, Italy, in 1941. He graduated cum laude in Electrical Engineering from the University of Bologna in 1964. Since 1975, he has been Professor of Automatic Control in that University. He has held visiting positions at various academic institutions, which include the University of California at Berkeley, the University of Florida at Gainesville, the MIT in Boston. He served the Italian scientific community as President of GRIS (Italian Group of Researchers on Computer and Systems) in the period 1981–1982, and as President of CIRA (Italian Inter-university Center of Research on Automatica) during the period 1989–1992. In 1986, he promoted the constitution of LAR (Lab of Automation and Robotics) at the Department of Electronics Computer and Systems, University of Bologna. In 1990 he launched the Center for distance-teaching technologies of the same University. From 1993 to 1996 he was the co-coordinator of ERNET (European Robotics Network) in the framework of the Human Capital and Mobility programme of the European Union. He was an elected member of the Administrative Council of EUAC (European Union Control Association) for the 2-term 1997–2002. Since 1994, he has been a permanent member of the Italian delegation at the IFAC (International Federation of Automatic Control).

His main research interests are in control systems design, fault detection and diagnosis, and robotic manipulation. He is author or co-author of more than 160 technical and scientific publications. He is author or co-author of five books on applied mathematics, digital control systems, and system identification and simulation. Professor Bonivento is also active in industrial control applications, at present with specific interest in automatic machinery for packaging and automotive control systems. Professor Bonivento is Director of DEIS (Department of Electronics Computer Science and Systems, University of Bologna) since 2000, a senior member of IEEE, and a member of the Adcom of EURON (European Robotics Research Network) since 2000. Since November 2002, he is the coordinator of the newly constituted research center CASY (Center on Complex Automated Systems) of DEIS.

Alberto Isidori graduated in Electrical Engineering from the University of Rome in 1965. Since 1975, he has been Professor of Automatic Control in this University. Since 1989, he is also affiliated with Washington University in St. Louis. Since 2001 he is also affiliated with CASY at the University of Bologna. He has held visiting positions at various academic/research institutions which include the University of Illinois at Urbana-Champaign, the University of California at Berkeley, the ETH in Zürich and the NASA-Langley research center. His research interests are primarily focused on mathematical control theory and control engineering. He is the author of several books, including \( \{ \text{cm Nonlinear Control Systems} \} \) (Springer, Berlin, 3rd ed., 1995) and \( \{ \text{cm Nonlinear Control Systems II} \} \) (Springer, Berlin, 1999). He is author of more than 200 technical publications, for a large part on the subject of nonlinear feedback design.

He is a Fellow of IEEE (1986). He received the G.S.Axelby Outstanding Paper Award in 1981 and in 1990 and the Automatica Best Paper Award in 1991. In 1996, he received from IFAC the “Georgio Quazza Medal” for “pioneering and fundamental contributions to the theory of nonlinear feedback control”. In 2000 he was awarded the first “Ktesibios Award” from the Mediterranean Control Association. In 2001, received the “Bode Lecture Prize” from the Control Systems Society of IEEE.
Lorenzo Marconi was born in Rimini, Italy, in 1970. He graduated in 1995 in Electrical Engineering from the University of Bologna. Since 1995, he has been with the Department of Electronics, Computer Science and Systems at University of Bologna, where he obtained his Ph.D. degree in March 1998. Since 1999 he has been a Assistant Professor in the same Department. He has held visiting positions at various academic/research institutions which include the Washington University at St. Louis, the Imperial College at London, the Ohio State University at Columbus Ohio, the Universite Paris-Sud, the Mittag-Leffler Institute. His current research interests include nonlinear control, output regulation, geometric approach and fault detection.

Andrea Paoli was born in Fano, Italy in 1975. He received the Laurea degree in Computer Science Engineering from the University of Bologna in 2000. He is currently a Ph.D. student at the Dipartimento di Elettronica, Informatica e Sistemistica at the University of Bologna, Italy. His current research interests focus on Fault Tolerant Control and Fault Diagnosis for complex distributed systems and on discrete event systems. His theoretical background includes also nonlinear control and output regulation using geometric approach.